GENERALIZED CLASS OF SYNTHETIC ESTIMATORS IN SMALL DOMAINS - A SIMULATION STUDY

K. K. PANDEY

AIM & ACT Deptt., Banasthali Vidyapith, Tonk, Rajasthan - 304022, India

&

G.C. TIKKIWAL

Deptt. of Mathematics & Statistics, J.N.V. University, Jodhpur-342 011, India

ABSTRACT

This paper defines and discusses a generalized class of synthetic estimator for small domain, using auxiliary information, under simple random sampling and stratified random sampling schemes. The generalized class of synthetic estimator, among others, includes the simple, ratio and product synthetic estimators. The proposed class of synthetic estimators gives consistent estimators if the synthetic assumption holds. Further, it demonstrates the use of the generalized synthetic and ratio synthetic estimators for estimating crop acreage for small domain and also compares their relative performance with direct estimators, empirically, through a simulation study.

Key words: Simulation-cum-Regression (SICURE) model; Synthetic Estimation; Small Domain; Inspector land Revenue Circles (ILRCs); Simple Random Sampling Without Replacement (SRSWOR)-design; Timely Reporting Scheme (TRS); Absolute Relative Bias (ARB); Simulated relative standard error (Srse).

1. INTRODUCTION

The common feature of small area estimation problem is that when large-scale sample survey are designed to produce reliable estimates at the national or state level; generally they do not provide estimates of adequate precision at lower levels like District, Tehsil / County, and Inspector land Revenue Circle. This is because the sample sizes at the lower level are generally insufficient to provide reliable estimates using traditional estimators.

Therefore, the need was felt to develop alternative estimators to provide small area statistics using the data already collected through large-scale surveys. The traditional design based and alternative estimators are also termed, in the literature of small area estimation, respectively as direct and indirect estimators.

The indirect estimators are based on methods which increase the effective sample size either by (i) simulating enough data through appropriate analysis of available data under appropriate modeling or (ii) by using data from other domains and /or time periods through models that assume similarities across domain and /or time periods. The only known method so far belonging to category (i) is SICURE- modeling [TIKKIWAL. (1993)].The other methods of estimation like Synthetic, Composite, and Generalized Regression belong to category (ii).Among these the synthetic estimators are used for small area estimation, mainly because of its simplicity, applicability to general sampling design and potential to increase accuracy in estimation. However, if the implicit model assumption of similarities across domain and /or time period fails, the synthetic estimator may be badly design biased. GONZALEZ (1973), GONZALEZ and WAKESBERG (1973), GHANGURDE AND SINGH (1977, 78) among others study the synthetic estimator based on auxiliary variables viz. the ratio synthetic estimator. These studies show that synthetic estimators provide reliable estimates to some extent.

In this paper we define a generalized class of synthetic estimators, using auxiliary information, under simple random sampling and stratified random sampling schemes. The generalized class of synthetic estimators, among others, includes the simple, ratio and product synthetic estimators. Further, we demonstrate the use of estimators belonging to the generalized class for estimating crop acreage for small domains and also compare their relative performance with the corresponding direct estimators, empirically, through a simulation study.

2. NOTATIONS

Suppose that a finite population U = (1, ... , i, ... , N) is divided into 'A' non overlapping small domains U_a of size N_a (a = 1, ... , A) for which estimates are required. We denote the characteristic under study by 'y'. We further assume that the auxiliary information is available and denote this by 'x'. A random sample s of size n is selected through Simple Random Sampling Without Replacement (SRSWOR) design from population U such that n_a units in the sample 's' comes from small domain U_a (a = 1, ..., A). It may be noted that if there is Simple Random Sampling With Replacement (SRSWR) design, it can be dealt with similarly.

Consequently,

$$\sum_{a=1}^{A} N_a = N \quad and \quad \sum_{a=1}^{A} n_a = n\,.$$

We denote the various population and sample means for characteristics Z = X, Y by

$\bar{Z}$ = mean of the population based on N observations.

$\bar{Z}_a$ = population mean of domain 'a' based on N_a observations.

$\bar{z}$ = mean of the sample 's' based on n observations.

$\bar{z}_a$ = sample mean of domain 'a' based on n_a observations.

Also, the various mean squares and coefficient of variations of the population 'U' for characteristics Z are denoted by

$$S_z^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(z_i - \bar{Z}\right)^2, \qquad C_z = \frac{S_z}{\bar{Z}}$$

The coefficient of covariance between X and Y is denoted by

$$C_{xy} = \frac{S_{xy}}{\bar{X}\bar{Y}}$$

where,

$$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right)$$

The corresponding various mean squares and coefficient of variations of small domains U_a are denoted by

$$S_{z_a}^2 = \frac{1}{N_a - 1} \sum_{i=1}^{N_a} \left(Z_{a_i} - \overline{Z_a} \right)^2 , \; C_{z_a} = \frac{S_{z_a}}{\overline{Z}_a} \quad \text{and } C_{x_a y_a} = \frac{S_{x_a y_a}}{\overline{X}_a \overline{Y}_a}$$

Where,

$$S_{x_a y_a} = \frac{1}{N_a - 1} \sum_{i=1}^{N_a} \left(y_{a_i} - \overline{Y}_a \right) \left(x_{a_i} - \overline{X}_a \right)$$

and z_{ai} ($a = 1, \ldots , A$ and $i = 1, \ldots , N_a$) denote the i-th observation of the small domain 'a' for the characteristic Z = X, Y.

3. GENERALIZED CLASS OF SYNTHETIC ESTIMATORS

We define a generalized class of synthetic estimators of population mean $\overline{Y}_a$, based on the auxiliary variable 'x' defined as follows:

$$\overline{y}_{syn,a} = \overline{y} \left(\frac{\overline{x}}{\overline{X}_a} \right)^{\beta} \qquad \ldots (3.1)$$

Where, β is a suitably chosen constant.

The above estimator $\overline{y}_{syn,a}$ may be heavily biased unless the following assumption is satisfied.

$$\overline{Y}_a \left(\overline{X}_a \right)^{\beta} \cong \overline{Y} \left(\overline{X} \right)^{\beta} \qquad \ldots (3.2)$$

It may be noted that if there is strict equality in the above relation; then the estimator given in Eq. (3.1) is a consistent estimator. This is so as the estimator reduces to $\overline{Y}_a$ when $n_a = N_a$ for all $a \in A$.

Remark 3.1

If β =0, then the estimator given in (3.1) reduces to simple-synthetic estimator

$$\overline{y}_{syn,s,a} = \overline{y}$$

and the corresponding assumption given in Eq. (3.2) reduces to

$$\overline{Y}_a \cong \overline{Y}$$

Remark 3.2

If β =-1, then the estimator given in (3.1) reduces to ratio-synthetic estimator

$$\overline{y}_{syn,r,a} = \left(\frac{\overline{y}}{\overline{x}}\right)\overline{X}_a$$

and the assumption given in Eq. (3.2) reduces to

$$\frac{\overline{Y}_a}{\overline{X}_a} \cong \frac{\overline{Y}}{\overline{X}}$$

Remark 3.3

Further If β=1, then generalized synthetic estimator reduces to product-synthetic estimator

$$\overline{y}_{syn,p,a} = \frac{\overline{y}\,\overline{x}}{\overline{X}_a}$$

and the assumption given in Eq. (3.2) reduces to

$$\overline{Y}_a\overline{X}_a \cong \overline{Y}\,\overline{X}$$

4. DESIGN BIAS AND MEAN SQUARE ERROR OF GENERALIZED SYNTHETIC ESTIMATOR

In order to obtain bias and mean square error of generalized synthetic estimator $\overline{y}_{syn,a}$,

let $\overline{y} = \overline{Y}(1+\varepsilon_1)$; $\overline{x} = \overline{X}(1+\varepsilon_2)$

So that $E(\varepsilon_1) = E(\varepsilon_2) = 0$ and

$$E(\varepsilon_1^2) = \frac{N-n}{Nn}C_y^2, \qquad E(\varepsilon_2^2) = \frac{N-n}{Nn}C_x^2 \quad \text{and} \quad E(\varepsilon_1\varepsilon_2) = \frac{N-n}{Nn}C_{xy} \ .$$

The $\overline{y}_{syn,a}$ can be expressed as

$$\overline{y}_{syn,a} = \overline{Y}\left(\frac{\overline{X}}{\overline{X}_a}\right)^{\beta}(1+\varepsilon_1)(1+\varepsilon_2)^{\beta} \ .$$

Assuming that $|\varepsilon_2| < 1$

$$\bar{y}_{syn,a} = \bar{Y}\left(\frac{\bar{X}}{\bar{X}_a}\right)^{\beta}(1+\beta\varepsilon_2+\frac{\beta(\beta-1)}{2}\varepsilon_2^2+\varepsilon_1+\beta\varepsilon_1\varepsilon_2+....)$$

Therefore

$$E(\bar{y}_{syn,a}) = \bar{Y}\left(\frac{\bar{X}}{\bar{X}_a}\right)^{\beta}\left[1+\frac{N-n}{Nn}\left(\frac{\beta(\beta-1)}{2}C_x^2+\beta C_{xy}\right)\right]$$

Assuming further that the contribution of terms involving powers in ε_1 and ε_2 higher than the second to the value of $E(\bar{y}_{syn,a})$ is negligible, and the design bias of $\bar{y}_{syn,a}$ is given by

$$B(\bar{y}_{syn,a}) = \bar{Y}\left(\frac{\bar{X}}{\bar{X}_a}\right)^{\beta}\left[1+\frac{N-n}{Nn}\left(\frac{\beta(\beta-1)}{2}C_x^2+\beta C_{xy}\right)\right]-\bar{Y}_a \quad .... (4.1)$$

If the synthetic assumption given in (3.2) satisfies than above expression reduces to

$$B(\bar{y}_{syn,a}) = \bar{Y}\left(\frac{N-n}{Nn}\right)\left[\left(\frac{1}{2}C_x^2\right)(\beta^2-\beta)+\beta C_{xy}\right]$$

And further design bias is zero either if $\beta = 0$ or $\beta = 1-2\frac{C_{xy}}{C_x^2}$ .The mean square error of $\bar{y}_{syn,a}$ is given by

$$MSE(\bar{y}_{syn,a}) = E(\bar{y}_{syn,a}-\bar{Y}_a)^2$$

$$= \bar{Y}^2\left(\frac{\bar{X}}{\bar{X}_a}\right)^{2\beta}\left[1+\frac{N-n}{Nn}\left\{(2\beta^2-\beta)C_x^2+C_y^2+4\beta C_{xy}\right\}\right]$$

$$-2\bar{Y}_a\bar{Y}\left(\frac{\bar{X}}{\bar{X}_a}\right)^{\beta}\left[1+\frac{N-n}{Nn}\left(\frac{\beta(\beta-1)}{2}C_x^2+\beta C_{xy}\right)\right]+\bar{Y}_a^2 \quad .... (4.2)$$

The suitable value of β is the one for which $MSE(\bar{y}_{syn,a})$ is minimum. So minimizing the $MSE(\bar{y}_{syn,a})$ with respect to β, gives simplified expression for β, if $\bar{X}_a \cong \bar{X}$ as follows

$$\beta = \frac{\bar{Y}(C_x^2 - 4C_{xy}) - 2\bar{Y}_a\left(\frac{C_x^2}{2} - C_{xy}\right)}{(4\bar{Y}C_x^2 - 2\bar{Y}_a C_x^2)} \qquad \text{…. (4.3)}$$

If the synthetic assumption given in (3.2) is satisfied then the above expression reduces to

$$MSE(\bar{y}_{syn,a}) = \frac{N-n}{Nn}\bar{Y}_a^2(\beta^2 C_x^2 + C_y^2 + 2\beta C_{xy}) \qquad \text{…. (4.4)}$$

And the value of β for which this expression of $MSE(\bar{y}_{syn,a})$ minimizes, is given by

$$\beta = \frac{-C_{xy}}{C_x^2} \quad .$$

5. GENERALIZED CLASS OF SYNTHETIC ESTIMATOR UNDER STRATIFIED RANDOM SAMPLING

Suppose that the finite population U=(1,…….,i,…….,N) is divided into 'A' non-overlapping domains $U_{\bullet a}$, of size $N_{\bullet a}$ (a=1,….,A), for which estimates are required as discussed in Section 2. The population is also divide along a second dimension into 'H' non-overlapping categories (called groups) $U_{h\bullet}$ of size $N_{h\bullet}$ (h=1,….., H). As a result, the population is cross classified into HA cells, U_{ha} of respective sizes N_{ha}. Consequently,

$$N = \sum_{h=1}^{h} N_{h\bullet} = \sum_{a=1}^{A} N_{\bullet a} = \sum_{h=1}^{H}\sum_{a=1}^{A} N_{ha} \qquad \text{… (5.1)}$$

We assume that N_{ha} are known from a previous census or other reliable sources. Further, we assume that simple random samples of predetermined size $n_{h\bullet}$ (h=1,….., H) are selected from group h such that $\sum_{h=1}^{H} n_{h\bullet} = n$. That is, n is size of the random sample selected using stratified random sampling. Also let $n_{\bullet a}$ and n_{ha} (a =1,….,A; h =1,….., H) are the units of the sample that belongs to domain $U_{\bullet a}$ and cell (h, a). So $n_{\bullet a}$ and n_{ha} are random.

Denoting y_{ha_i} (i=1,….., N_{ha}), the i-th observation of the characteristic under study of the cell (h, a), we define various population and sample means as follows, using capital letters for population means and small letters for sample means.

$$\overline{Y}_{a\bullet} = \frac{1}{N_{\bullet a}} \sum_{h=1}^{H} N_{ha} \overline{Y}_{ha} \quad ; \qquad \text{Population mean of small area ‘a’.}$$

Where

$$\overline{Y}_{ha} = \frac{1}{N_{ha}} \sum_{i=1}^{N_{ha}} y_{ha_i}$$

$$\overline{y}_{\bullet a} = \frac{1}{n_{\bullet a}} \sum_{h=1}^{H} n_{ha} \overline{y}_{ha} \quad ; \qquad \text{Sample mean of small area ‘a’.}$$

Where

$$\overline{y}_{ha} = \frac{1}{n_{ha}} \sum_{i=1}^{n_{ha}} y_{ha_i}$$

$$\overline{Y}_{h\bullet} = \frac{1}{N_{h\bullet}} \sum_{a=1}^{A} N_{ha} \overline{Y}_{ha} \quad ; \qquad \text{Population mean of the } h^{th} \text{ group.}$$

$$\overline{y}_{h\bullet} = \frac{1}{n_{h\bullet}} \sum_{a=1}^{A} n_{ha} \overline{y}_{ha} \quad ; \qquad \text{Sample mean of the } h^{th} \text{ group.}$$

Similar notations are used, for various means for auxiliary characteristic x, just replacing ‘y’ with ‘x’ symbol. Then following (3.1), a generalized synthetic estimator under stratification is defined as follows.

$$\overline{y}_{s,syn,a} = \sum_{h=1}^{H} w_{ha} \overline{y}_{h\bullet} \left(\frac{\overline{x}_{h\bullet}}{\overline{X}_{ha}} \right)^{\beta} \qquad \text{… (5.2)}$$

Under the synthetic assumption

$$\overline{Y}_{ha}(\overline{X}_{ha})^{\beta} \cong \overline{Y}_{h\bullet}(\overline{X}_{h\bullet})^{\beta} \qquad \ldots (5.3)$$

Where, $w_{ha} = \dfrac{N_{ha}}{N_{\bullet a}}$

Now the design bias of $\overline{y}_{s,syn,a}$ is given by

$$B(\overline{y}_{s,syn,a}) = E(\overline{y}_{s,syn,a}) - \overline{Y}_a$$

$$= \sum_{h=1}^{H}\left[w_{ha}\overline{Y}_{h\bullet}\left(\frac{\overline{X}_{h\bullet}}{\overline{X}_{ha}}\right)^{\beta}\left\{1+\frac{N_{h\bullet}-n_{h\bullet}}{N_{h\bullet}n_{h\bullet}}\left(\frac{\beta(\beta-1)}{2}C_{hx}^{2}+\beta C_{hxy}\right)\right\} - \overline{Y}_{ha}\right] \qquad \ldots (5.4)$$

If the synthetic assumption (5.3) satisfies, the expression of the bias reduces to

$$B(\overline{y}_{s,syn,a}) = \sum_{h=1}^{H} w_{ha}\left[\overline{Y}_{ha}\left\{\frac{N_{h\bullet}-n_{h\bullet}}{N_{h\bullet}n_{h\bullet}}\left(\frac{\beta(\beta-1)}{2}C_{hx}^{2}+\beta C_{hxy}\right)\right\}\right] \qquad \ldots (5.5)$$

Similarly, expression of mean square error of the $\overline{y}_{s,syn,a}$ can be obtained, using the expression (4.2) of $MSE(\overline{y}_{s,syn,a})$.

Remark 5.1

For $\beta = 0$, the estimator (5.2) reduces to

$$\overline{y}'_{s,syn,a} = \sum_{h=1}^{H} w_{ha}\,\overline{y}_{h\bullet} \quad , \quad \text{This in turn gives}$$

$$\hat{T}_{s,syn,a} = \sum_{h=1}^{H} N_{ha}\,\overline{y}_{h\bullet}$$

The estimator of population total 'T_a' of small area 'a', discussed by SARANDAL [1984, Eq. (3.1), p.625].

Also the expression (5.4) of bias for $\beta = 0$ reduces to

$$B(\overline{y}'_{s,syn,a}) = \sum_{h=1}^{H} w_{ha}\,(\overline{Y}_{h\bullet} - \overline{Y}_{ha})$$

Which in turn gives expression of bias of $\hat{T}_{s,syn,a}$

$$B(\hat{T}_{s,syn,a}) = \sum_{h=1}^{H} N_{ha}\,(\overline{Y}_{h\bullet} - \overline{Y}_{ha})$$

[see SARNDAL 1984, Eq. (6.1), p. 628].

Remark 5.2

Cassel et al. (1987) uses the above estimator $\overline{y}'_{s,syn,a}$ to provide estimate of unemployment at municipal level in Swedish Labor Force survey. The performance of this estimator proves to be better than the corrected synthetic estimator.

Remark 5.3

The ratio and product synthetic estimator under stratification can be obtain for different values of β as obtained, for simple estimator $\overline{y}'_{s,syn,a}$ by substituting $\beta = 0$. For example, for $\beta = -1$, generalized estimator (5.2) reduces to

$$\overline{y}''_{s,syn,a} = \sum_{h=1}^{H} w_{ha}\left(\frac{\overline{y}_{h\bullet}}{\overline{x}_{h\bullet}}\right)\overline{X}_{ha}\,.$$

This estimator is currently in use to provide improved estimates of States income in U.S.A. [see Schaible (1996), pp. 28-57].

6. CROP ACREAGE ESTIMATION FOR SMALL DOMAIN- A SIMULATION STUDY

In this section we demonstrate the use of the generalized synthetic and ratio synthetic estimators to obtain crop acreage estimates for small domain and also compare their relative performance with the corresponding direct estimators empirically, through a simulation study. This we do by taking up the state of Rajasthan, one of the states in India, for case study.

6.1 EXISTING METHODOLOGY FOR ESTIMATION

In order to improve timelines and quality of crop acreage statistics, a scheme known as Timely Reporting Scheme (TRS) has been in vogue since early seventies in most of the States of India. The TRS has the objective of providing quick and reliable estimates of crop acreage statistics and there-by production of the principle crops during each agricultural season. Under the scheme the Patwari (Village Accountant) is required to collect acreage statistics on a priority basis in a 20 percent sample of villages, selected by stratified linear systematic sampling design taking Tehsil (a sub-division of the District) as a stratum. These statistics are further used to provide state level estimates using direct estimators viz. Unbiased (based on sample mean) and ratio estimators.

The performance of both the estimators in the State of Rajasthan, like in other states, is satisfactory at state level, as the sampling error is within 5 percent. However, the sampling error of both the estimators increases considerably, when they are used for estimating acreage statistics of various principle crops even at district level, what to speak of levels lower than a district. For example, the sampling error of direct ratio estimator for Kharif crops (the crop sown in June – July and harvested in October – November every year) of Jodhpur district (of Rajasthan State) for the agricultural season 1991-92 varies approximately between 6 to 68 percent. Therefore, there is need to use indirect estimators at district and lower levels for decentralized planning and other purposes like crop insurance, bank loan to formers.

6.2 DETAILS OF THE SIMULATION STUDY

For collection of revenue and administrative purposes, the State of Rajasthan, like most of the other states of India, is divided into a number of districts. Further, each district is divided into a number of Tehsils and each Tehsil is also divided into a number of Inspector Land Revenue Circles (ILRCs). Each ILRC consists of a number of villages. For the present study, we take ILRCs as small domains.

In the simulation study, we undertake the problem of crop acreage estimation for all Inspector Land Revenue Circles (ILRCs) of Jodhpur Tehsil of Rajasthan. They are seven in number and these ILRCs contain respectively 29, 44, 32, 30, 33, 40 and 44 villages. These ILRCs are small domains from the TRS point of view. The crop under consideration is Bajra (Indian corn or millet) for the agriculture season 1993-94. The Bajra crop acreage for agriculture season 1992-93 is taken as the auxiliary characteristic x. The various information regarding the ILRCs of Jodhpur Tehsil are provided in the Table 6.2.1.

We consider the following estimators of population total 'Ta' of small domain 'a' for a = 1, 2,..., 7

Direct Estimators:

Direct ratio Estimator $$t_{1,a} = N_a\left(\frac{\bar{y}_a}{\bar{x}_a}\right)\bar{X}_a$$

Direct general estimator $$t_{2,a} = N_a\bar{y}_a\left(\frac{\bar{x}_a}{\bar{X}_a}\right)^{\beta}$$

Indirect Estimators:

Ratio synthetic Estimator $$t_{3,a} = N_a\left(\frac{\bar{y}}{\bar{x}}\right)\bar{X}_a$$

Generalized synthetic Estimator $$t_{4,a} = N_a\bar{y}\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta}$$

Table 6.2.1

Total Area (Irrigated and Unirrigated) under Bajara Crop in Inspector Land Revenue Circles (ILRCs) of Jodhpur Tehsil for Agricultural seasons 1992-93 and 1993-94.

S.No	ILRCs of Jodhpur Tehsil	No. of villages in ILRc	Total Area (Irr. +U.I.) under the crop Bajra in 1992-93	Total Area (Irr. +U.I.) under the crop Bajra in 1993-94
1.	Jodhpur (1)	29	07799.5899	05696.5000
2.	Karu(2)	44	21209.5880	15699.6656
3.	Dhundhada(3)	32	19019.0288	16476.4863
4.	Bisalpur(4)	30	15153.9248	14269.0000
5.	Luni(5)	33	19570.1323	16821.4508
6.	Dhava(6)	40	25940.0979	25075.5000
7.	Jajiwal Kalan (7)	44	18007.4120	15875.0000
	Total	252	126699.7737	109913.6027

Before simulation, we first examine the following assumption, given earlier in (3.1), for ratio synthetic estimator $t_{3,a}$ and generalized synthetic estimator $t_{4,a}$ with respect to the seven domain under study:

$$\bar{Y}_a\left(\bar{X}_a\right)^{\beta} \cong \bar{Y}\left(\bar{X}\right)^{\beta}$$

For $\beta=-1$ for estimator $t_{3,a}$ and for the optimum value of β given in (4.4) for estimator $t_{4,a}$, the Tables 6.2.2 and 6.2.3 provide below absolute differences between $\bar{Y}_a/\bar{X}_a$ and $\bar{Y}/\bar{X}$, and between $\bar{Y}_a\left(\bar{X}_a\right)^{\beta}$ and $\bar{Y}\left(\bar{X}\right)^{\beta}$ for all the seven ILRCs respectively. From the examination of these tables we note in this study that both the assumption closely meet in ILRCs 3, 5 and 7, deviate moderately in ILRCs 4 and 6, but deviate considerably in ILRCs 1 and 2.

TABLE 6.2.2

Absolute Differences (Relative) under Synthetic Assumption of Synthetic Ratio Estimator for Various ILRCs.

ILRC	$\bar{Y}_a/\bar{X}_a$	$\bar{Y}/\bar{X}$	$\left[\left\|(\bar{Y}_a/\bar{X}_a)-(\bar{Y}/\bar{X})\right\|\div\left(\bar{Y}_a/\bar{X}_a\right)\right]\times 100$
(1)	.7303	.8675	18.17
(2)	.7402	.8675	17.19
(3)	.8663	.8675	0.13
(4)	.9416	.8675	7.86
(5)	.8595	.8675	0.91
(6)	.9666	.8675	10.25
(7)	.8815	.8675	1.58

TABLE 6.2.3

Absolute Differences Under Synthetic Assumption of Generalized Synthetic Estimator for Various ILRCs.

ILRC	$\bar{Y}_a(\bar{X}_a)^{\beta_2}$	$\bar{Y}(\bar{X})^{\beta_2}$	$\left[\left\|\{\bar{Y}_a(\bar{X}_a)^{\beta_2} - \bar{Y}(\bar{X})^{\beta_2}\}\right\| \div \bar{Y}_a(\bar{X}_a)^{\beta_2}\right] \times 100$
(1)	3.31157	4.6578	40.65232
(2)	2.11349	2.4947	18.03699
(3)	0.77584	0.7791	0.42019
(4)	1.23143	1.1343	7.887578
(5)	0.8136	0.82231	1.070551
(6)	0.14251	0.13789	3.241878
(7)	2.40008	2.44412	1.834939

Now taking village as sampling units for simulation purposes and otherwise, 500 independent simple random samples for each size of 25,50,63,76 and 88 are selected from the population of 252 village of Jodhpur Tehsil. Then, to assess the relative performance of the estimator under consideration, their Absolute Relative Bias(ARB) and Simulated relative standard error (Srse) or simply coefficient of variation are calculated for each ILRCs as follows:

$$ARB(t_{k,a}) = \frac{\left| \frac{1}{500}\sum_{s=1}^{500} t_{k,a}^{s} - T_a \right|}{T_a} \times 100 \qquad \text{... (6.2.1)}$$

and

$$Srse(t_{k,a}) = \frac{\sqrt{ASE(t_{k,a})}}{E(t_{k,a})} \times 100 \qquad \ldots (6.2.2)$$

where

$$ASE(t_{k,a}) = \frac{1}{500} \sum_{s=1}^{500} (t_{k,a}^{s} - T_a)^2 \qquad \ldots (6.2.3)$$

for k = 1, 2,…, 7 and a = 1,2, ..., 7.

6.3 RESULTS

We present the results of ARB and Srse in Table (6.3.1) only $n = 50$ (a sample of 20 present villages, as presently adopted in TRS) as the findings from other tables are similar.

For assessing the relative performance of the various estimators, we have to adopt some rule of thumb. Here we adopt the rule that at the ILRCs level, an estimator should not have Srse more than 10 % and bias more than 5%. We note from the table that none of the estimators satisfy the rule in ILRCs 1 and 2. This is happening because, in these circles, there is considerable deviation from the synthetic assumption, as observed earlier.

Table 6.3.1

Simulated relative standard error (in %) and Absolute Relative Bias (in %) for various ILRCs under SRSWOR scheme, for n = 50

	ILRC						
Estimator	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\hat{T}_{1,a}$	37.27 (0.21)	17.46 (2.28)	8.51 (0.76)	16.29 (0.13)	12.73 (2.41)	12.28 (0.32)	15.29 (2.78)
$\hat{T}_{2,a}$	18.55 (0.96)	18.32 (1.50)	6.56 (0.12)	15.43 (0.18)	1.27 (1.12)	13.68 (0.54)	1.34 (0.61)
$\hat{T}_{3,a}$	40.54 (39.67)	21.00 (19.84)	5.96 (0.11)	10.17 (8.68)	5.95 (0.18)	12.14 (11.03)	7.16 (3.97)
$\hat{T}_{4,a}$	19.11 (17.90)	20.67 (19.50)	5.71 (0.72)	10.11 (8.66)	5.71 (0.05)	8.43 (4.60)	5.85 (1.02)

In ILRCs 4 and 6, where the assumption deviate moderately, $t_{4,a}$ alone satisfies the rule to some extent. In ILRCs 3, 5 and 7, where the synthetic assumption closely meet, both $t_{3,a}$ and $t_{4,a}$ satisfy the rule but $t_{4,a}$'s performance is slightly better than $t_{3,a}$.

From the above analysis it is clear that if the synthetic estimators do not deviate considerably from their corresponding synthetic assumption then, performance of the synthetic estimators $t_{3,a}$ and $t_{4,a}$, based on a sample of 20 present villages (as presently being taken under TRS), is satisfactory at the level of ILRCs. Therefore, these estimators are also likely to perform better both at Tehsil and district levels. When the synthetic estimators deviate considerably from their corresponding synthetic assumptions then we should look for other types of estimators such as those obtained through the SICURE MODEL [TIKKIWAL, 1993] and assess their relative performance through studies of the kind, in series, over some years for crop acreage estimation.

Note: The figures shown in parentheses are the Absolute Relative Biases in percentage.

REFERENCES

Casell, C.M. Kristiansson, E.K., Raback, G., Wahlstrom (1987). Using model- based estimation to improve the estimate of unemployment on a regional level in the Swedish Labor Force survey. Small area Statistics, John Wiley and Sons, New York.

Ghangurde, P.D. And Singh, M.P (1977). Synthetic estimates in periodic household surveys. Survey methodology, 3, 151-181.

Ghangurde, P.D. And Singh, M.P. (1978). Evaluation of efficiency of synthetic estimates. Proceeding of the social statistical section of the american statistical association , 53-61.

Gonzalez, M.E. (1973). Use and evaluation of synthetic estimates. Proceedings of the social statistical section of American statistical association, 33-36.

Gonzalez, M.E. and Waksberg, J. (1973). Estimation of the error of synthetic estimates.paper presented at first meeting of the International association of survey statisticians , Vienna, Austria , 18-25, august 1973.

Sarndal, C.E. (1984). Design- consistent versus model- dependent estimators for small domains. Journal of the American statistical association , 79, 624-631.

Schaible, W.W. (1996). Indirect Estimators in U.S. Federal programs. Research Monograph, springer- verlag.

Tikkiwal, B.D. (1993). Modelling through survey data for small domains. Proceedings of International Scientific conference on small area statistics and survey Design (held in September, 1992 at Warsaw, Poland).

Tikkiwal, B.D. (1985). The Theory of t-classes of linear estimators, its generic value and its impact on teaching. A paper at 50th biennial session of International Stat. Institute at Amsterdam, Nether land.

Tikkiwal, B.D. (1986). Lecture notes on advanced theory of sample surveys. A revised version of earlier monograph (1970), Dept. of Statistics, University of Rajasthan Library.

Tikkiwal, B.D. (1989). Simulation-cum-regression approach to small area estimation. Abstract, Inst. Math. Stat. Bulletin, 6, 575-576.

Tikkiwal, B.D. (1993). Modelling through survey data for small domains. Proceedings of International Scientific conference on small area statistics and survey Design (held in September, 1992 at Warsaw, Poland).

Tikkiwal, B.D. and Tikkiwal, G.C. (1991). Sampling strategies in survey. The role of the theory of T-classes and computers. Symposium Volume, Indian Agric. Stat. Research Institute, New Delhi, 287-296.

Tikkiwal, B.D. and Tikkiwal, G.C. (1994). Small area estimation. An invited paper presented by second author in person before the 81th session of Indian science association held at Jaipur from January 3-8, 1994.

Tikkiwal, G.C. (1997). On estimation of crop area statistics for small domains. An invited paper, presented in person before the 2nd annual, conference of Information theory and application and national symposium in statistics with focus on applications in agriculture and allied sciences, held at CCS Haryana Agricultural University, Hissar from February 7-9, 1997. Paper is under publication in the proceedings.

Tikkiwal, G.C. and Pandey, K.K.; On Synthetic and Composite Estimators for Small Area Estimation under Lahiri – Midzuno Sampling Scheme. Statistics in Transition- new series, April 2007, vol. 8. No. 1, pp. 111-123.

Tikkiwal, G.C. and Pandey, K.K. (2007). A Generalized Class of Synthetic Estimators with Application to Crop Acreage Estimation for Small Domains. A paper presented in International Conference at Mumbai from May 31 to June 2, 2007.

Tikkiwal, G.C. and Pandey, K.K. (2006). Generalized Class of Synthetic Estimators for Small Area under Systematic Sampling Scheme. A paper presented in National Conference at Saurastra University, Gujrat form November 11-13, 2006.