CHAPTER-3

MODEL SELECTION IN SHALE GAS RESERVOIRS

3.1 RESERVOIR MODEL:

In this study, a Dual Porosity Model has been proposed for constructing a reservoir simulation model for shale gas reservoir. Same as the normal Dual Porosity Model, in this model, the shale gas reservoir contains Matrix and natural fractures along with hydraulic fractures. As the natural fractures are not uniform throughout the shale reservoir, the assumption of gas flow from matrix to natural fracture will not be applicable in all the cases. Based on intensity of natural fracture the effective matrix permeability will be enhanced. In order to overcome this situation, In present work matrix pores and natural fractures as single porous zone and the hydraulic fractures as the second porous zone have been assumed Based on aforesaid assumption, a reservoir model has been developed. The following are the assumptions that are considered while developing this model.

1) The flows of gas from the matrix to hydraulic fracture and then from hydraulic fracture to horizontal well bore.
2) Only single phase flow (only gas flow) in the matrix.
3) Two phase flow (Gas + Water) in the hydraulic fracture is assumed.
4) No gas is flown directly from the matrix to the horizontal well bore.
5) The only source of gas for wellbore is the hydraulic fracture.

The pictorial representation of the model is shown in Figure 3.1

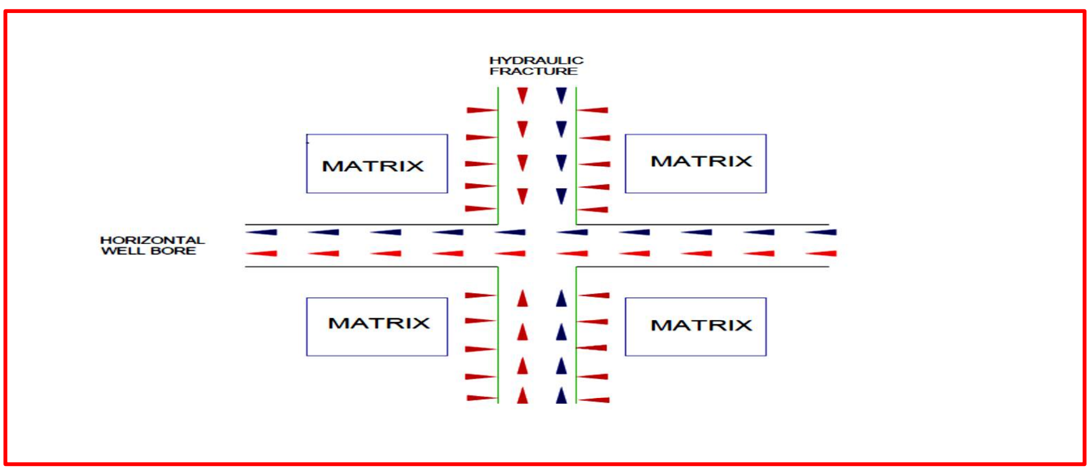


Figure 3.1: Reservoir Model representing the flow of gas from matrix to hydraulic fractures and from hydraulic fracture to well bore.

3.2 Mass Balance Equation for Gas Flow in the Matrix:

The pictorial representation of gas flow in the matrix is as in Figure 3.2

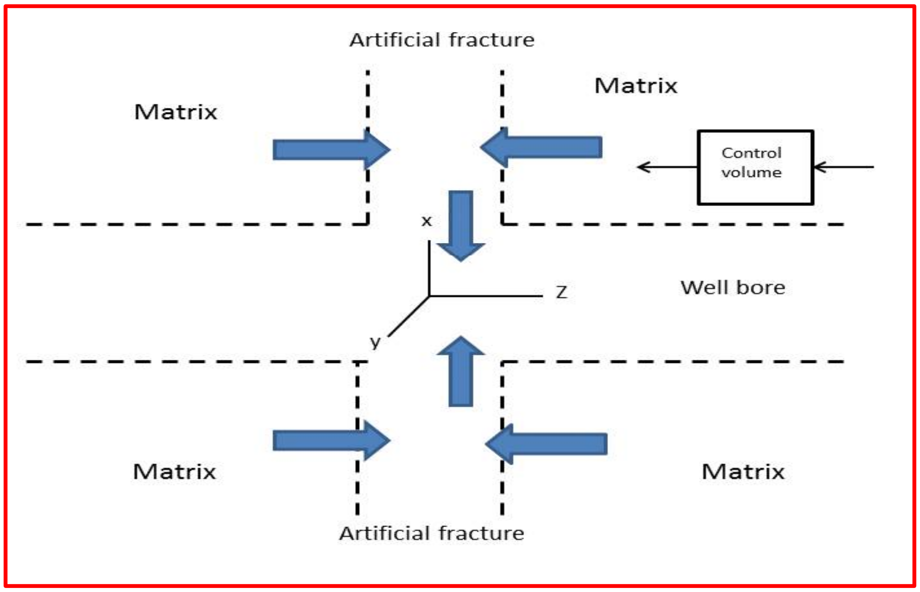


Figure 3.2: Schematic representation of gas flow from matrix to fracture.

3.2.1 Assumptions:-

1) Flow of water in the matrix is negligible.
2) Considering the process as Isothermal Process i.e. Constant Temperature.
3) Gas flow through matrix has been considered to be of Darcy type with incorporation of Klinkenberg effect for slippage or non-viscous flow or molecular flow through tiny pores.
4) Since natural fractures are believed to be discrete, they do not significantly contribute to gas flow on regional scale by themselves. Their effect can be incorporated in the matrix effective permeability or porosity.

3.2.2 Mass Balance Equation:

In the present work the free gas in the matrix and the adsorbed gas in the matrix for developing mass balance equation have been considered.

The control volume of the matrix is: - $\Delta x * \Delta y * \Delta z$.(Negative sign indicates the gas flow is in negative x,y and z directions).

Mass balance equation is given as,

Mass of free gas in – Mass of free gas out+ Mass of gas desorbed/generated=Mass rate of change of gas (free & adsorbed) in the control volume.

⇨ $-\left(v_{g_{mx}}\rho_{g_m|_{x+\Delta x}}\Delta y\Delta z\right)-\left(-v_{g_{mx}}\rho_{g_m|_x}\Delta y\Delta z\right)+\left(-v_{g_{mx}}\rho_{g_m|_{y+\Delta y}}\Delta x\Delta z\right)-$

$\left(-v_{g_{my}}\rho_{g_m|_y}\Delta x\Delta z\right)+\left(-v_{g_{mz}}\rho_{g_m|_{z+\Delta z}}\Delta x\Delta y\right)-\left(-v_{g_{mz}}\rho_{g_m|_z}\Delta x\Delta y\right)+\frac{\Delta\left((\Delta x\Delta y\Delta z.(1-\emptyset_m).\rho_m.\rho_{gs}.V_d\right)}{\Delta t}$

$$= \frac{\Delta\left((\Delta x\Delta y\Delta z.(1-\emptyset_m).\rho_m.\rho_{gs}.V_a+\Delta x\Delta y\Delta z.\emptyset_m.S_{g_m}\rho_{g_m}\right)}{\Delta t}. \text{ --→ } \mathbf{1.}$$

Where, $V_a = V_L - V_d$

V_a= Remaining adsorbed gas volume at standard conditions.

V_d = Desorbed gas volume.

V_L = Langmuir Volume i.e. maximum volume of gas adsorbed per unit mass of rock in volume.

Now, dividing equation 1 by$(\Delta x\Delta y\Delta z)$, we got

⇨ $$\frac{(-v_{g_{mx}}\,\rho_g|_{x+\Delta x}+v_{g_{mx}}\,\rho_g|_x)}{\Delta x}+\frac{\left(-v_{g_{my}}\,\rho_g|_{y+\Delta y}+v_{g_{my}}\,\rho_g|_y\right)}{\Delta y}+\frac{(-v_{g_{mz}}\,\rho_g|_{z+\Delta z}+v_{g_{mz}}\,\rho_g|_z)}{\Delta z}+$$
$$\frac{\Delta\left(((1-\emptyset_m).\rho_m.\rho_{gs}.V_d\right)}{\Delta t}=\frac{\Delta\left(((1-\emptyset_m).\rho_m.\rho_{gs}.V_a+\emptyset_m.S_{g_m}\rho_{g_m}\right)}{\Delta t} \text{ --------------------------------------→ } \mathbf{2.}$$

Now taking limit for Δx, Δy, Δz and Δt --→ 0, the equation can be written as

⇨ $$\frac{\partial(v_{g_{mx}}\rho_{gm})}{\partial x}+\frac{\partial\left(v_{g_{my}}\rho_{gm}\right)}{\partial y}+\frac{\partial(v_{g_{mz}}\rho_{gm})}{\partial z}=\frac{\partial[((1-\emptyset_m).\rho_m.\rho_{gs}.V_a+\emptyset_m.S_{g_m}\rho_{g_m}]}{\partial t}-$$
$$\frac{\partial[((1-\emptyset_m).\rho_m.\rho_{gs}.V_d]}{\partial t} \text{ --→ } \mathbf{3.}$$

Now considering Darcy's Law

$$q_{gx}=\frac{-k\,A}{\mu}.\frac{dP}{dx}.$$

For x-direction,

⇨ $\frac{Gas\ Volume}{time} = \frac{(-k.\Delta y.\Delta z)}{\mu} \frac{\partial(P_m)}{\partial x}$.

⇨ $\frac{(\Delta x.\Delta y.\Delta z)}{t} = \frac{(-k.\Delta y.\Delta z)}{\mu} \frac{\partial(P_m)}{\partial x}$.

⇨ $\frac{\Delta x}{t} = \frac{(-k)}{\mu} \frac{\partial(P_m)}{\partial x}$. --→ **4.**

But velocity $(v_{g_{mx}}) = \frac{Displacement}{time}$

⇨ $time\ (t) = \frac{Displacement(\Delta x)}{v_{g_{mx}}}$

Now substituting $time\ (t)$ in equation 4, I got

⇨ $v_{g_{mx}} = \frac{-k_m\ k_{rg}}{\mu_g} \frac{\partial(P_m)}{\partial x}$ ----------------→ for x-direction.

⇨ $v_{g_{my}} = \frac{-k_m\ k_{rg}}{\mu_g} \frac{\partial(P_m)}{\partial y}$ ----------------→ for y-direction.

⇨ $v_{g_{mz}} = \frac{-k_m\ k_{rg}}{\mu_g} \frac{\partial(P_m)}{\partial z}$ ----------------→ for z-direction.

Where,

k_m= *Effective matrix permeability.*

$k_m = k_\infty \left(1 + \frac{b}{P_m}\right)$ *; including klinkenberg Effect.*

k_∞= *Equivalent liquid permeability of matrix.*

S_g= *Gas Saturation in rock pore = constant.(* S_g*=1).*

ρ_{gs}= *Standard gas density = constant.*

ρ_m= *Rock density = constant.*

V_d= *standard volume of desorbed gas per unit rock mass.*

Now substituting all the above terms in equation 3, we get

$$\Rightarrow \frac{\partial\left(\frac{-k_m\ k_{rg}}{\mu_g}\frac{\partial(P_m)}{\partial x}\rho_{gm}\right)}{\partial x} + \frac{\partial\left(\frac{-k_m\ k_{rg}}{\mu_g}\frac{\partial(P_m)}{\partial y}\rho_{gm}\right)}{\partial y} + \frac{\partial\left(\frac{-k_m\ k_{rg}}{\mu_g}\frac{\partial(P_m)}{\partial z}\rho_{gm}\right)}{\partial z} = \frac{\partial[((1-\emptyset_m).\rho_m.\rho_{gs}.V_a+\emptyset_m.S_{gm}\rho_{gm}]}{\partial t} - \frac{\partial[((1-\emptyset_m).\rho_m.\rho_{gs}.V_d]}{\partial t}.$$

$$\Rightarrow \left[\frac{\partial\left(\frac{-k_m\ k_{rg}}{\mu_g}\frac{\partial(P_m)}{\partial x}\rho_{gm}\right)}{\partial x} + \frac{\partial\left(\frac{-k_m\ k_{rg}}{\mu_g}\frac{\partial(P_m)}{\partial y}\rho_{gm}\right)}{\partial y} + \frac{\partial\left(\frac{-k_m\ k_{rg}}{\mu_g}\frac{\partial(P_m)}{\partial z}\rho_{gm}\right)}{\partial z}\right] = S_{g_m}\frac{\partial(\emptyset_m\,\rho_{gm})}{\partial t} + \rho_m\rho_{gs}\frac{\partial((1-\emptyset_m)(V_L-V_d))}{\partial t} - \rho_m\rho_{gs}\frac{\partial((1-\emptyset_m)(V_d))}{\partial t} \text{ ---------→ } \mathbf{5.}$$

From energy of state,

Formation volume factor (B_g) = $\frac{\rho_{gsc}}{\alpha_c\rho_{gm}}$

Where,

α_c = *Volume Conversion Factor = 5.6145* $\frac{BTU}{ft^3}$.

ρ_{gsc} = *Density of gas at standard conditions.*

ρ_{gm} = *Density of gas in the Matrix.*

$$\Rightarrow \rho_{gm} = \frac{\rho_{gsc}}{\alpha_{cB_g}}$$

Now, substituting ρ_{gm} in Eqn 5, we get

$$\Rightarrow \left[\frac{\partial\left(\frac{k_m\ k_{rg}}{\mu_g}\frac{\partial(P_m)}{\partial x}\ \frac{\rho_{gsc}}{\alpha_{cB_g}}\right)}{\partial x} + \frac{\partial\left(\frac{k_m\ k_{rg}}{\mu_g}\frac{\partial(P_m)}{\partial y}\ \frac{\rho_{gsc}}{\alpha_{cB_g}}\right)}{\partial y} + \frac{\partial\left(\frac{k_m\ k_{rg}}{\mu_g}\frac{\partial(P_m)}{\partial z}\frac{\rho_{gsc}}{\alpha_{cB_g}}\right)}{\partial z}\right] = -S_{g_m}\frac{\partial\left(\emptyset_m\frac{\rho_{gsc}}{\alpha_{cB_g}}\right)}{\partial t} - \rho_m\rho_{gs}\frac{\partial((1-\emptyset_m)(V_L-V_d))}{\partial t} + \rho_m\rho_{gs}\frac{\partial((1-\emptyset_m)(V_d))}{\partial t} \text{ ---------→ } \mathbf{6.}$$

Now multiplying Eqn 6 with bulk volume i.e. Δx.Δy.Δz

$$\Rightarrow \left[\frac{\partial\left(\frac{k_m\,\beta_c\,k_{rg}A_x}{\mu_g}\frac{\partial(P_m)}{\partial x}\;\frac{\rho_{gsc}}{\alpha_{cB_g}}\right)}{\partial x}\Delta x + \frac{\partial\left(\frac{k_m\,\beta_c\,k_{rg}A_y}{\mu_g}\frac{\partial(P_m)}{\partial y}\;\frac{\rho_{gsc}}{\alpha_{cB_g}}\right)}{\partial y}\Delta y + \right.$$

$$\left.\frac{\partial\left(\frac{k_m\,\beta_c\,k_{rg}A_z}{\mu_g}\frac{\partial(P_m)}{\partial z}\frac{\rho_{gsc}}{\alpha_{cB_g}}\right)}{\partial z}\Delta z\right] = -V_b S_{g_m}\frac{\partial\left(\emptyset_m\frac{\rho_{gsc}}{\alpha_{cB_g}}\right)}{\partial t} - V_b\rho_m\rho_{gs}\frac{\partial\left((1-\emptyset_m)(V_L-V_d)\right)}{\partial t} +$$

$$V_b\rho_m\rho_{gs}\frac{\partial\left((1-\emptyset_m)(V_d)\right)}{\partial t} \text{---} \rightarrow \mathbf{7.}$$

Where β_c = Transmissibility conversion factor = 1.127 $\frac{scf}{D-psi}$.

Now consider the R.H.S in equation 7,

$$\Rightarrow \frac{\partial\left((1-\emptyset_m)(V_L-V_d)\right)}{\partial t} = \frac{\partial\left((1-\emptyset_m)(V_a)\right)}{\partial t}$$

$$= V_a\frac{\partial\left((1-\emptyset_m)\right)}{\partial t} + (1-\emptyset_m)\frac{\partial\left((V_a)\right)}{\partial t}$$

$$= \left[V_a\frac{\partial\left((1-\emptyset_m)\right)}{\partial p_m} + (1-\emptyset_m)\frac{\partial\left((V_a)\right)}{\partial p_m}\right]\frac{\partial p_m}{\partial t} \text{------------------------} \rightarrow \mathbf{8.}$$

$$\Rightarrow Consider, \frac{\partial\left((V_a)\right)}{\partial p_m}$$

$$\frac{\partial\left((V_a)\right)}{\partial p_m} = \frac{\partial\left(\left(\frac{V_L P_m}{P_L+P_m}\right)\right)}{\partial p_m}$$

$$\Rightarrow V_L\frac{\partial\left(\left(\frac{P_m}{P_L+P_m}\right)\right)}{\partial p_m} = V_L\frac{\partial\left((P_m.(P_L+P_m)^{-1})\right)}{\partial p_m}$$

$$\Rightarrow V_L\left[(P_L+P_m)^{-1} + P_m\frac{\partial\left((P_L+P_m)^{-1}\right)}{\partial P_m}\right]$$

$$\Rightarrow V_L\left[(P_L+P_m)^{-1} + P_m\frac{-1}{((P_L+P_m)^2)}\right]$$

$$\Rightarrow V_L\left[\frac{1}{(P_L+P_m)} - \frac{P_m}{(P_L+P_m)^2}\right]$$

$$\Rightarrow \frac{P_L V_L}{(P_L+P_m)^2}.$$

$$\Rightarrow \frac{\partial\left((V_a)\right)}{\partial p_m} = \frac{P_L V_L}{(P_L+P_m)^2}$$

From literature, $\emptyset_m = \emptyset_o e^{c_m\,(P_m - P_o)}$.

⇨ $\frac{\partial \emptyset_m}{\partial P_m} = \emptyset_o e^{c_m (P_m - P_o)}\, c_m.$

$= \emptyset_m . c_m$

⇨ $\frac{\partial (1-\emptyset_m)}{\partial P_m} = - \frac{\partial \emptyset_m}{\partial P_m} = - \emptyset_m . c_m$

Now substituting $\frac{\partial((V_a))}{\partial p_m}$ and $\frac{\partial(1-\emptyset_m)}{\partial P_m}$ in equation 8, we get

⇨ $\frac{\partial((1-\emptyset_m)(V_L - V_d))}{\partial t} = \left[V_a(-\emptyset_m . c_m) + (1-\emptyset_m)\frac{P_L V_L}{(P_L+P_m)^2}\ \right]\frac{\partial p_m}{\partial t}$ --------------------------→ **9.**

Considering the term

$$\frac{\partial((1-\emptyset_m)(V_d))}{\partial t}$$

⇨ $\frac{\partial((1-\emptyset_m)(V_d))}{\partial t} = \frac{\partial((1-\emptyset_m)(V_d))}{\partial p_m}\frac{\partial p_m}{\partial t}.$

$= \left\{V_d \frac{\partial((1-\emptyset_m))}{\partial p_m} + (1-\emptyset_m)\ \frac{\partial((V_d))}{\partial p_m}\right\}\frac{\partial p_m}{\partial t}$ --→**10.**

$V_d = V_L - V_a$

$= V_L - \frac{V_L P_m}{P_L+P_m}$

$= \frac{V_L P_L}{P_L+P_m}$

$\frac{\partial((V_d))}{\partial p_m} = \frac{\partial\left(\left(\frac{V_L P_L}{P_L+P_m}\right)\right)}{\partial p_m}$

$= \frac{\partial\left(\left((P_L V_L)\ (P_L+P_m)^{-1}\right)\right)}{\partial p_m}$

$= - \frac{P_L V_L}{(P_L+P_m)^2}$

From equation 10,

$$\frac{\partial((1-\emptyset_m)(V_d))}{\partial p_m}\frac{\partial p_m}{\partial t} = \left\{V_d(-\emptyset_m c_m) + (1-\emptyset_m)\left(-\frac{P_L V_L}{(P_L+P_m)^2}\right)\right\}\frac{\partial p_m}{\partial t}.$$

$$= \left\{-\frac{P_L V_L}{(P_L+P_m)^2} + \emptyset_m \frac{P_L V_L}{(P_L+P_m)^2} - V_d \emptyset_m c_m\right\}\frac{\partial p_m}{\partial t}$$

Substituting $\frac{\partial((1-\emptyset_m)(V_L - V_d))}{\partial t}$ and $\frac{\partial((1-\emptyset_m)(V_d))}{\partial t}$ in Eqn 7, We get

$$\Rightarrow \left[\frac{\partial\left(\frac{k_m \beta_c k_{rg} A_x}{\mu_g}\frac{\partial(P_m)}{\partial x}\frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\partial x}\Delta x + \frac{\partial\left(\frac{k_m \beta_c k_{rg} A_y}{\mu_g}\frac{\partial(P_m)}{\partial y}\frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\partial y}\Delta y + \frac{\partial\left(\frac{k_m \beta_c k_{rg} A_z}{\mu_g}\frac{\partial(P_m)}{\partial z}\frac{\rho_{gsc}}{\alpha_c B_g}\right)}{\partial z}\Delta z\right] = \left[\frac{-V_b S_{gm} \rho_{gsc} \emptyset_m c_m}{\alpha_c B_g} - V_b \rho_m \rho_{gsc} \emptyset_m c_m (V_d - V_a) + \frac{2 P_L V_L V_b (1-\emptyset_m)}{(P_L+P)^2}\right]\frac{\partial p_m}{\partial t} \text{ ---------------} \rightarrow \mathbf{11.}$$

The above equation represents the flow of gas in the matrix.

The above equation is a non-linear PDE equation which has to be solved to determine the variation of pressure with time.

The following relations are used for calculatingK_m, μ_g, ρ_{gsc},B_g,C_m,V_a and V_d.(which will vary with respect to pressure)

Klinkenberg Effect (K_m) = $K_{darcy}\left(1 + \frac{b_k}{P_m}\right)$

Where, Klinkenberg Coefficient (b_k)= $12.639 K_{darcy}{}^{-0.33}$.

Viscosity (μ_g) is calculated by using the following correlation

$$\ln\left(T_{pr}\frac{\mu_g}{\mu_1}\right) = a_0 + a_1 P_{pr} + a_2 P_{pr}{}^2 + a_3 P_{pr}{}^3 + T_{pr}\left(a_4 + a_5 P_{pr} + a_6 P_{pr}{}^2 + a_7 P_{pr}{}^3\right) + T_{pr}{}^2\left(a_8 + a_9 P_{pr} + a_{10} P_{pr}{}^2 + a_{11} P_{pr}{}^3\right) + T_{pr}{}^3\left(a_{12} + a_{13} P_{pr} + a_{14} P_{pr}{}^2 + a_{15} P_{pr}{}^3\right)$$

Density of gas at standard conditions (ρ_{gsc})

$$\Rightarrow \quad \rho_{gsc} = \frac{P_{sc} M_a}{Z_{sc} R T_{sc}}$$

Gas formation volume factor (B_g):

$$B_g = 0.02827 \frac{Z\,T}{P_m}.$$

Now, for solving the equation 11 for the entire reservoir, the reservoir is divided into several blocks i.e matrix blocks. In this model we have divided the entire reservoir into 9*9*9 3D reservoir as shown in Figure 3.3.

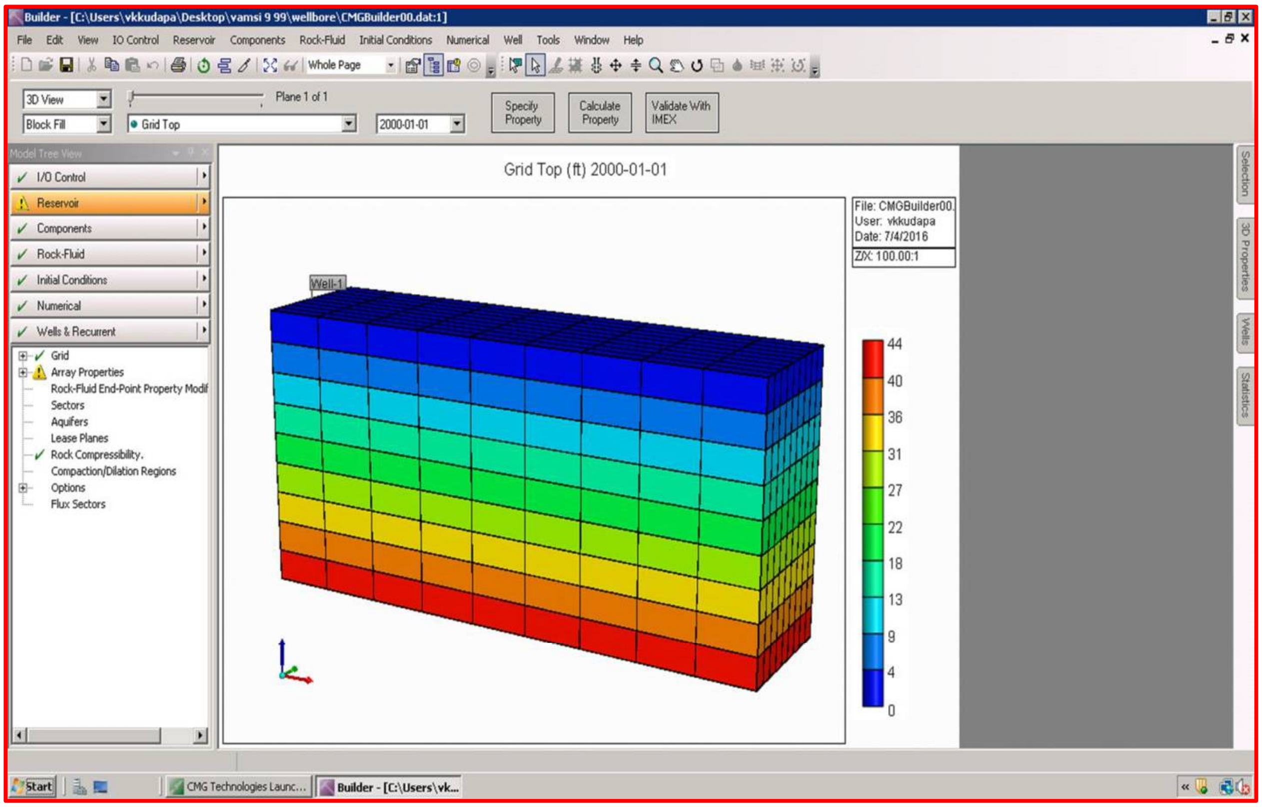


Figure 3.3: Schematic view of the 3D Reservoir.

3.2.3 <u>Discretization Method:</u>

The developed equation representing the gas flow in the matrix blocks is a nonlinear partial differential equation (PDE). For discretization of this nonlinear PDE, finite difference method has been used.

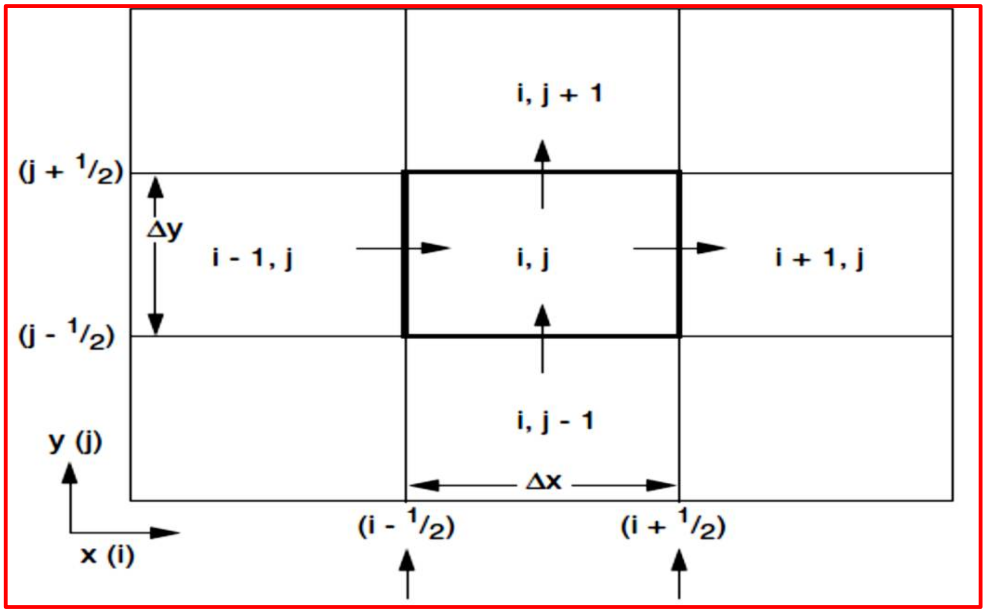


Figure 3.4: Discretization and notation indication for a 3D pressure equation

By applying finite difference method, the equation 11 can be written as

$$\Rightarrow \left[\left(\frac{k_m\,\beta_c\,k_{rg}A_x}{\mu_g}\ \frac{\rho_{gsc}}{\alpha_{cB_g}}\ \frac{1}{\Delta x}\right)_{i+\frac{1}{2},j,k}\left(P^{n+1}_{i+1,j,k}-P^{n+1}_{i,j,k}\right)-\left(\frac{k_m\,\beta_c\,k_{rg}A_x}{\mu_g}\ \frac{\rho_{gsc}}{\alpha_{cB_g}}\ \frac{1}{\Delta x}\right)_{i-\frac{1}{2},j,k}\left(P^{n+1}_{i,j,k}-P^{n+1}_{i-1,j,k}\right)\right]+\left[\left(\frac{k_m\,\beta_c\,k_{rg}A_y}{\mu_g}\ \frac{\rho_{gsc}}{\alpha_{cB_g}}\ \frac{1}{\Delta y}\right)_{i,j+\frac{1}{2},k}\left(P^{n+1}_{i,j+1,k}-P^{n+1}_{i,j,k}\right)-\left(\frac{k_m\,\beta_c\,k_{rg}A_y}{\mu_g}\ \frac{\rho_{gsc}}{\alpha_{cB_g}}\ \frac{1}{\Delta y}\right)_{i,j-\frac{1}{2},k}\left(P^{n+1}_{i,j,k}-P^{n+1}_{i,j-1,k}\right)\right]+\left[\left(\frac{k_m\,\beta_c\,k_{rg}A_z}{\mu_g}\ \frac{\rho_{gsc}}{\alpha_{cB_g}}\ \frac{1}{\Delta z}\right)_{i,j,k+\frac{1}{2}}\left(P^{n+1}_{i,j,k+1}-P^{n+1}_{i,j,k}\right)-\left(\frac{k_m\,\beta_c\,k_{rg}A_z}{\mu_g}\ \frac{\rho_{gsc}}{\alpha_{cB_g}}\ \frac{1}{\Delta z}\right)_{i,j,k-\frac{1}{2}}\left(P^{n+1}_{i,j,k}-P^{n+1}_{i,j,k-1}\right)\right]=$$

$$\left[\frac{-V_bS_{gm}\rho_{gsc}\emptyset_m c_m}{\alpha_c B_g}-V_b\rho_m\rho_{gsc}\emptyset_m c_m\left(V_d-V_a\right)+\frac{2P_LV_LV_b(1-\emptyset_m)}{(P_L+P)^2}\right]\frac{\left(P^{n+1}_{i,j,k}-P^{n}_{i,j,k}\right)}{\Delta t} \text{ ------------------}\rightarrow \textbf{12.}$$

As we have chosen the spatial discretization terms at new time interval i.e. (n+1), the applied finite difference method can be considered has implicit finite difference method.

$Considering\ transmisibility(T_{gx}) = \frac{\beta_c\,k_{rg}}{\mu_g}\ \frac{\rho_{gsc}}{\alpha_{cB_g}}$ in all-direction

Eqn 12 becomes

$$\Rightarrow \left[\left(\frac{K_m A_x T_{gx}}{\Delta x}\right)_{i+\frac{1}{2},j,k}\left(P^{n+1}_{i+1,j,k} - P^{n+1}_{i,j,k}\right) - \left(\frac{K_m A_x T_{gx}}{\Delta x}\right)_{i-\frac{1}{2},j,k}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i-1,j,k}\right)\right] + \left[\left(\frac{K_m A_y T_{gy}}{\Delta y}\right)_{i,j+\frac{1}{2},k}\left(P^{n+1}_{i,j+1,k} - P^{n+1}_{i,j,k}\right) - \left(\frac{K_m A_y T_{gy}}{\Delta y}\right)_{i,j-\frac{1}{2},k}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i,j-1,k}\right)\right] + \left[\left(\frac{K_m A_z T_{gz}}{\Delta z}\right)_{i,j,k+\frac{1}{2}}\left(P^{n+1}_{i,j,k+1} - P^{n+1}_{i,j,k}\right) - \left(\frac{K_m A_z T_{gz}}{\Delta z}\right)_{i,j,k-\frac{1}{2}}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i,j,k-1}\right)\right] = \left[\frac{-V_b S_{gm}\rho_{gsc}\emptyset_m c_m}{\alpha_c B_g} - V_b\rho_m\rho_{gsc}\emptyset_m c_m\,(V_d - V_a) + \frac{2P_L V_L V_b(1-\emptyset_m)}{(P_L+P)^2}\right]\frac{\left(P^{n+1}_{i,j,k} - P^{n}_{i,j,k}\right)}{\Delta t}$$

-----------------------------→ **13.**

Writing eqn 13 in the following form

$$\Rightarrow \lambda_{g_{i+\frac{1}{2},j,k}}\left(P^{n+1}_{i+1,j,k} - P^{n+1}_{i,j,k}\right) - \lambda_{g_{i-\frac{1}{2},j,k}}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i-1,j,k}\right) + \lambda_{g_{i,j+\frac{1}{2},k}}\left(P^{n+1}_{i,j+1,k} - P^{n+1}_{i,j,k}\right) - \lambda_{g_{i,j-\frac{1}{2},k}}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i,j-1,k}\right) + \lambda_{g_{i,j,k+\frac{1}{2}}}\left(P^{n+1}_{i,j,k+1} - P^{n+1}_{i,j,k}\right) - \lambda_{g_{i,j,k-\frac{1}{2}}}\left(P^{n+1}_{i,j,k} - P^{n+1}_{i,j,k-1}\right) = X_{i,j,k}\left(P^{n+1}_{i,j,k} - P^{n}_{i,j,k}\right)$$

-----------------→ **14.**

where,

$$\lambda_{g_{i+\frac{1}{2},j,k}} = \left(\frac{K_m A_x T_{gx}}{\Delta x}\right)_{i+\frac{1}{2},j,k}$$

$$\lambda_{g_{i-\frac{1}{2},j,k}} = \left(\frac{K_m A_x T_{gx}}{\Delta x}\right)_{i-\frac{1}{2},j,k}$$

$$\lambda_{g_{i,j+\frac{1}{2},k}} = \left(\frac{K_m A_y T_{gy}}{\Delta y}\right)_{i,j+\frac{1}{2},k}$$

$$\lambda_{g_{i,j-\frac{1}{2},k}} = \left(\frac{K_m A_y T_{gy}}{\Delta y}\right)_{i,j-\frac{1}{2},k}.$$

$$\lambda_{g_{i,j,k+\frac{1}{2}}} = \left(\frac{K_m A_z T_{gz}}{\Delta z}\right)_{i,j,k+\frac{1}{2}}$$

$$\lambda_{g_{i,j,k-\frac{1}{2}}} = \left(\frac{K_m A_z T_{gz}}{\Delta z}\right)_{i,j,k-\frac{1}{2}}$$

$$X_{i,j,k} = \left[\frac{-V_b S_{gm}\rho_{gsc}\emptyset_m c_m}{\alpha_c B_g} - V_b\rho_m\rho_{gsc}\emptyset_m c_m\,(V_d - V_a) + \frac{2P_L V_L V_b(1-\emptyset_m)}{(P_L+P)^2}\right]$$

Eqn 14 can be written as

$$B_{i,j,k}P_{i,j,k-1}^{n+1} + S_{i,j,k}P_{i,j-1,k}^{n+1} + W_{i,j,k}P_{i-1,j,k}^{n+1} + C_{i,j,k}P_{i,j,k}^{n+1} + E_{i,j,k}P_{i+1,j,k}^{n+1} + N_{i,j,k}P_{i,j+1,k}^{n+1} + A_{i,j,k}P_{i,j,k+1}^{n+1} = Q_{i,j,k} \quad \text{------}\rightarrow \mathbf{15.}$$

Where,

$B_{i,j,k} = \lambda_{g_{i,j,k-\frac{1}{2}}}.$

$S_{i,j,k} = \lambda_{g_{i,j-\frac{1}{2},k}}.$

$W_{i,j,k} = \lambda_{g_{i-\frac{1}{2},j,k}}$

$C_{i,j,k} = -\left[\lambda_{g_{i+\frac{1}{2},j,k}} + \lambda_{g_{i-\frac{1}{2},j,k}} + \lambda_{g_{i,j+\frac{1}{2},k}} + \lambda_{g_{i,j-\frac{1}{2},k}} + \lambda_{g_{i,j,k+\frac{1}{2}}} + \lambda_{g_{i,j,k-\frac{1}{2}}}. + X_{i,j,k}\right]$

$E_{i,j,k} = \lambda_{g_{i+\frac{1}{2},j,k}}.$

$N_{i,j,k} = \lambda_{g_{i,j+\frac{1}{2},k}}$

$A_{i,j,k} = \lambda_{g_{i,j,k+\frac{1}{2}}}$

$Q_{i,j,k} = \left(-X_{i,j,k}\right)P_{i,j,k}^{n}$

Equation 15 is applied for the entire reservoir in the $N_1*N_2*N_3$ 3D Reservoir. The obtained equations are solved by coding using JAVA.

In this case, the well bore is placed horizontally in the 5^{th} layer and the gas will flow from the adjacent matrixes to the wellbore. The schematic representation of this model is shown in Figure 3.5.

Figure 3.5: Schematic Representation of Wellbore in the 5th Layer from Top.

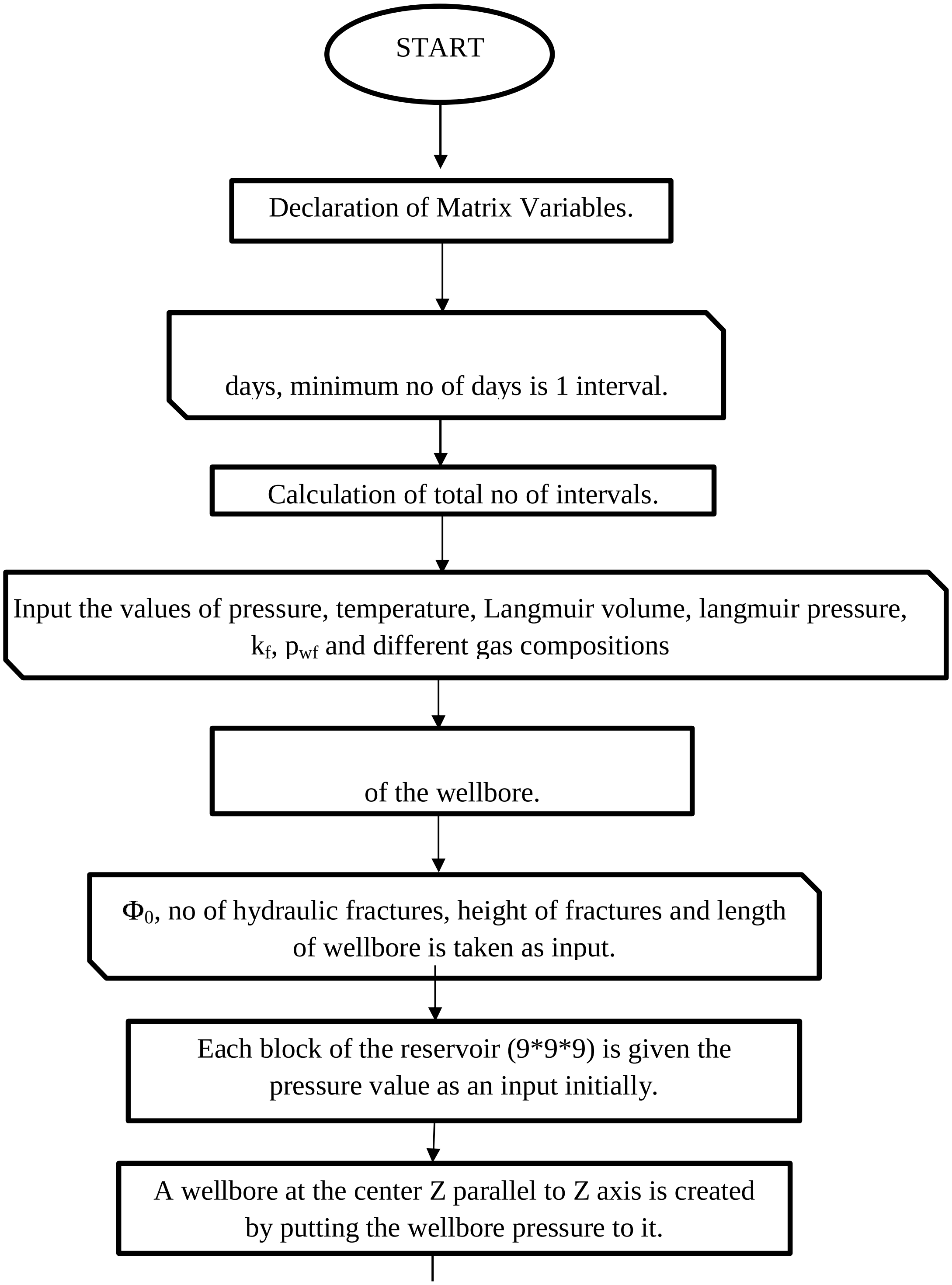
The detailed input properties of the shale reservoir are listed in the table 3.1

Table 3.1: Parameters used for determining the gas flow in the matrix.

Parameter	Value	Units
Matrix Dimensions.	1166*293*5.55	ft
Matrix Porosity.	0.07	
Matrix Permeability.	0.0002	mD
Reservoir Temperature.	240	^{0}F
Reservoir Thickness.	50	ft
Horizontal Wellbore. Length	9000	ft
Wellbore Diameter	1	ft
Wellbore Pressure	100	psi
Reservoir Pressure	3800	psi
Fracture Spacing	1	ft
Gas specific gravity	0.68	
Gas Composition:	$CH_4 = 0.85$	
	$CO_2 = 0.08$	
	$N_2 = 0.04$	
	H_2S= 0.03.	

3.3 **Algorithm:**

The developed nonlinear partial differential equation is compiled using JAVA for determining the pressure variation in the matrix blocks during gas production from shale reservoirs. Figure 3.6 represents the algorithm, used for solving the nonlinear PDE for gas flow inside the matrix.


START
Declaration of Matrix Variables.
days, minimum no of days is 1 interval.
Calculation of total no of intervals.
Input the values of pressure, temperature, Langmuir volume, langmuir pressure,
k_f, p_{wf} and different gas compositions
of the wellbore.
Φ_0, no of hydraulic fractures, height of fractures and length
of wellbore is taken as input.
Each block of the reservoir (9*9*9) is given the
pressure value as an input initially.
A wellbore at the center Z parallel to Z axis is created
by putting the wellbore pressure to it.

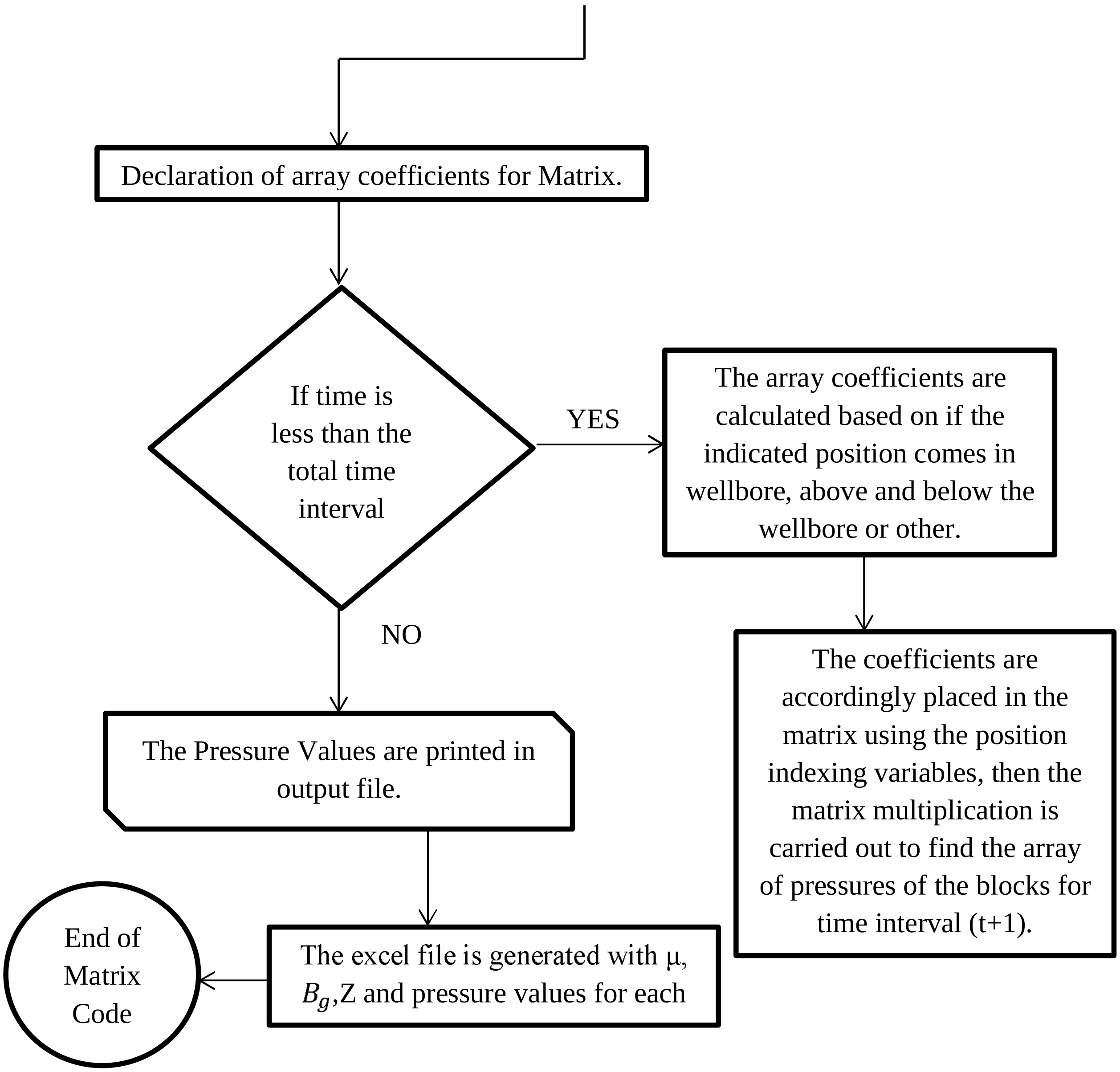


Figure 3.6: Algorithm representing the procedure for solving the gas flow in the matrix.

The derived nonlinear partial differential equation for flow of gas in the matrix is compiled using JAVA programming language and the code is attached in ANNEXURE-I.