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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, April 2018

Programme: B.Tech (Mechatronics) | **Semester – VIII**
Course Name: Optimization Techniques | **Max. Marks : 100**
Course Code: MEEL-432 | **Duration : 03 Hrs**
No. of page/s: 03

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

Section A
(Attempt all questions)

1.	Consider the following linear programming problem. Introduce slack variables to the given inequality constraints, and hence set up the initial simplex table. $$\begin{aligned} \max\ & 3x_1 + 5x_2 + 4x_3 \\ \text{subject to } & 2x_1 + 3x_2 \le 8 \\ & x_1 + 5x_3 \le 10 \\ & 3x_1 + 2x_2 + 4x_3 \le 15 \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$	**[4]**	**CO1**
2.	Find the dual of the following problem. $$\begin{aligned} \max\ & 2x_1 + 3x_2 + x_3 \\ \text{subject to } & 4x_1 + 3x_2 + x_3 = 6 \\ & x_1 + 2x_2 + 5x_3 = 4 \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$	**[4]**	**CO2**
3.	Consider the function of three variables given by $$f(x_1, x_2, x_3) = x_1^2 - x_1 - x_1x_2 + x_2^2 - x_2 + x_3^4 - 4x_3.$$ Compute the Hessian matrix, $H(x_1, x_2, x_3)$.	**[4]**	**CO4**
4.	Put the following problem in a standard (maximization) form. $$\begin{aligned} \min\ & 3x_1 - 4x_2 - x_3 \\ \text{subject to } & x_1 + 3x_2 - 4x_3 \le 12 \\ & 2x_1 - x_2 + x_3 \le 20 \\ & x_1 - 4x_2 - 5x_3 \ge 5 \end{aligned}$$ $x_1 \ge 0, x_2$ and x_3 are unrestricted in sign.	**[4]**	**CO1**

5.	Find the local maximum and minimum, if any, of the following function: $$f(x) = x^3 - 3x^2 + 3x - 1.$$	**[4]**	**CO4**
SECTION B **(Q6-Q8 are compulsory and Q9 has internal choice)**			
6.	Solve the following linear programming problem by Simplex method. $$\max 3x_1 + 9x_2$$ $$\text{subject to } x_1 + 4x_2 \leq 8$$ $$x_1 + 2x_2 \leq 4$$ $$x_1, x_2 \geq 0.$$	**[10]**	**CO1**
7.	Find the minimum of the function $$f(\lambda) = 0.65 - \frac{0.75}{1+\lambda^2} - 0.65\lambda \tan^{-1}\frac{1}{\lambda}$$ using Newton method with the starting point $\lambda_0 = 0.1$. Perform three iteration.	**[10]**	**CO3**
8.	Perform two iteration tables, to solve the following LPP by Big- M method $$\max 2x_1 + x_2 + 3x_3$$ $$\text{subject to } x_1 - 2x_2 + 3x_3 = 2$$ $$3x_1 + 2x_2 + 4x_3 = 1$$ $$x_1, x_2, x_3 \geq 0.$$	**[10]**	**CO1**
9.	Find the minimum of the function $f(x) = x(x - 1.5)$ in the interval $(0.0, 1.0)$ to within 10% of the exact value, by interval halving method. **OR** Find the minimum of the function $$f(\lambda) = \frac{\lambda}{\log_e \lambda}$$ using Secant method with the starting point $\lambda_0 = 0.1$. Perform three iterations.	**[10]**	**CO3**

SECTION C (Q10 is compulsory and Q11 has internal choice)			
10A.	Consider the function $$f(x_1, x_2) = x_1^3 + x_2^3 - 3x_1x_2.$$ a. Find the critical points for the function $f(x_1, x_2)$. b. Compute the Hessian matrix corresponding to each critical point. c. Find the local maximum and minimum, if any, using Hessian matrix.	**[10]**	**CO4**
10B.	Find the dual of the following problem. $\min 3x_1 + 2x_2$ subject to $7x_1 + 2x_2 \geq 30$ $5x_1 + 4x_2 \geq 20$ $2x_1 + 8x_2 \geq 16$ $x_1, x_2 \geq 0.$ Hence, set up initial table to solve the dual problem by Big-M method.	**[10]**	**CO2**
11.	Find the dual of the following problem. $\max x_1 + 6x_2$ subject to $x_1 + x_2 \geq 2$ $x_1 + 3x_2 \geq 3$ $x_1, x_2 \geq 0.$ Use graphical method to solve the primal and the dual, and show that the optimal values of the objective functions of the problems are equal. **OR** By solving dual of the LPP $\min 2x_1 + 2x_2$ subject to $2x_1 + 4x_2 \geq 1$ $x_1 + 2x_2 \geq 1$ $2x_1 + x_2 \geq 1$ $x_1, x_2 \geq 0,$ show that the optimal value of the primal problem is $\frac{4}{3}$.	**[20]**	**CO2**