Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme Name: B.Tech. (CivilE-Sz-Infra) **Semester : I**
Course Name : Mathematics I **Time : 03 hrs**
Course Code : MATH 1008 **Max. Marks : 100**
Nos. of page(s) : 2
Instructions: All questions are compulsory.

SECTION A

S. No.		Marks	CO
Q1	Show that two vectors are linearly dependent if and only if one vector is scalar multiple of the other vector.	**4**	**CO1**
Q2	Test the consistency for solution of the following system of equations. $x+y+z=3$ $x+2y+3z=4$ $x+4y+9z=6$	**4**	**CO1**
Q3	Find the volume of a rectangular parallelopiped with length, width and height as a, b and c, respectively.	**4**	**CO3**
Q4	Test for the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots\infty$	**4**	**CO4**
Q5	Find the interval of convergence for the power series of $\log(1+x)$.	**4**	**CO4**

SECTION B

Q6	Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the canonical form and specify the matrix of transformation.	**8**	**CO1**
Q7	Show that $\Gamma(n+1)=n\Gamma n$ for $n\in R, n>0$ and hence show that for a natural number, n, $\Gamma n=(n-1)!$.	**8**	**CO2**
Q8	Find the maximum value of $x^m y^n z^p$, when $x+y+z=a$ with m, n, p, a as constants.	**8**	**CO2**
Q9	If x increases at the rate of $2\,cm/sec$ at the instant when $x=3\,cm$ and $y=1\,cm$., at what rate must y be changing in order that the function $2xy-3x^2y$ shall be neither increasing nor decreasing ? **OR** If $x=r\cos\theta$ and $y=r\sin\theta$, verify that $\frac{\partial(x,y)}{\partial(r,\theta)}\frac{\partial(r,\theta)}{\partial(x,y)}=1$.	**8**	**CO2**

Q10	Evaluate the following integral over a parallelogram in the xy-plane with vertices $(1,0),(3,1),(2,2),(0,1)$ using the transformation $u=x+y$ and $v=x-2y$. $$\iint (x+y)^2\,dx\,dy$$ **OR** Change the order of the integration in the following and then integrate it. $$\int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} x^2\,dy\,dx$$	**8**	**CO3**
	SECTION-C		
Q11A	Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$.	**10**	**CO3**
Q11B	Obtain the Fourier series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$.	**10**	**CO4**
Q12A	A plate of the form of a quadrant of the ellipse $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$ is of small but varying thickness, the thickness at any point being proportional to the product of the distances of that point from the major and the minor axes. Find the co-ordinates of the centre of gravity of the plane. **OR** Find the mass of a lamina in the form of the cardioid $r=a(1+\cos\theta)$ whose density at any point varies as the square of its distance from the initial line.	**10**	**CO3**
Q12B	Show that the p-series $$\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots+\infty$$ is convergent for $p>1$ and divergent for $p\le 1$. **OR** Expand the function $f(x)$ given below as the Fourier *cosine* series. $$f(x)=\begin{cases} kx, if\ 0<x<\frac{l}{2} \\ k(l-x), if\ \frac{1}{2}<x<1 \end{cases}$$	**10**	**CO4**

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SECTION A

S. No.		Marks	CO
Q1	Show that the set of vectors $X=[1,2,-3,4], Y=[3,-1,2,1]$, $Z=[1,-5,8,-7]$ is linearly independent.	**4**	**CO1**
Q2	Define positive definite and negative definite real quadratic forms. Classify the real quadratic form, $X'AX$, as positive or negative definite, where $A=\begin{bmatrix}1 & 3\\ 0 & 2\end{bmatrix}$.	**4**	**CO1**
Q3	Find area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ using double integral.	**4**	**CO3**
Q4	Test for the convergence of the series $\frac{1}{4.7.10}+\frac{4}{7.10.13}+\frac{9}{10.13.16}+\ldots\infty$	**4**	**CO4**
Q5	Express $f(x)=x$ as half-range sine series in $0<x<2$.	**4**	**CO4**

SECTION B

Q6	Find the rank of the following matrix by reducing it to the Echelon form. $\begin{bmatrix}1 & 2 & 3 & 2\\ 2 & 3 & 5 & 1\\ 1 & 3 & 4 & 5\end{bmatrix}$	**8**	**CO1**
Q7	Let $\frac{a_0}{n+1}+\frac{a_1}{n}+\frac{a_2}{n-1}+\ldots+\frac{a_{n-1}}{2}+a_n=0$. Show that the function $a_0x^n+a_1x^{n-1}+\ldots+a_n$ vanishes at least once in $(0,1)$.	**8**	**CO2**
Q8	Find that maximum and minimum of $\sin x \sin y \sin(x+y); 0<x, y<\pi$.	**8**	**CO2**
Q9	If $u=f(x,y)$, where $x=r\cos\theta$ and $y=r\sin\theta$, prove that $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$ **OR**	**8**	**CO2**

	If $x=r\cos\theta$ and $y=r\sin\theta$, verify that $\frac{\partial(x,y)}{\partial(r,\theta)}\frac{\partial(r,\theta)}{\partial(x,y)}=1$		
Q10	Change the order of the integration in the following and then integrate it. $\int_0^a \int_{x^2/2}^{2a-x} xydy\,dx$ **OR** Transform the following double integral in polar coordinates and then evaluate. it. $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{1}{\sqrt{a^2-x^2-y^2}}dy\,dx$	**8**	**CO3**
	SECTION-C		
Q11A	Evaluate the following triple integral. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$	**10**	**CO3**
Q11B	Find a Fourier series to represent $x-x^2$ from $x=-\pi$ to $x=\pi$.	**10**	**CO4**
Q12A	Find the mass of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. The tetrahedron has the variable density $\rho=kxyz$. **OR** A solid is in the form of the positive octant of the sphere $x^2+y^2+z^2=a^2$. The density ρ at any point (x,y,z) is given by $\rho=\mu xyz$, where μ is a constant. Find the coordinates of the centre of gravity of the solid.	**10**	**CO3**
Q12B	Expand the function $f(x)$ given below as the Fourier *sine* series. $f(x)=\begin{cases}\frac{1}{4}-x, if\ 0<x<\frac{1}{2}\\ x-\frac{3}{4}, if\ \frac{1}{2}<x<1\end{cases}$ **OR** Test for the convergence of the following series.	**10**	**CO4**

	$\frac{1}{2}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\ldots\infty$		