Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Course: System Modeling and Identification (CSAI 7002) **Semester: I (2018-2019)**

Programme: M.Tech (A & RE – I)

Time: 03 hrs. **Max. Marks: 100**

Instructions: Attempt all questions from **Section A** (each carrying 4 marks); all questions from **Section B** (each carrying 8 marks) and all questions from **Section C** (carrying 20 marks)**.**

SECTION A

S. No.		Marks	CO
Q1	Classify the following partial differential equations $(i)\ 3u_{xx} + u_{xy} - 4u_{yy} + 3u_x = 0$ $(ii)\ u_{xx} - 6u_{xy} + 9u_{yy} - 17u_y = 0.$	**4**	**CO2**
Q2	Determine the value of y at $x = 0.1$ by Picard's method for only one approximation, given that $\frac{dy}{dx} = \frac{y-x}{y+x},\ y(0) = 1.$	**4**	**CO1**
Q3	Define the node and saddle point of a linear autonomous system with examples.	**4**	**CO3**
Q4	Determine the nature of the critical point $(0,0)$ of the system $\frac{dx}{dt} = -8x - 7y,\ \frac{dy}{dt} = 3x + 2y$ and determine whether or not the critical point is stable.	**4**	**CO3**
Q5	A frame ***F*** has been moved 10 units along the ***y***-axis and 5 units along the **z**-axis of the reference frame. Determine the new location of the frame, where $F = \begin{bmatrix} 0.527 & -0.574 & 0.628 & 5 \\ 0.369 & 0.819 & 0.439 & 3 \\ -0.766 & 0 & 0.643 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$	**4**	**CO5**

SECTION B

Q6	Solve the differential equation $\frac{d^2y}{dx^2} + y = x,\ x \in [0,2]$ with the boundary conditions $y(0) = 0, \&\ y(2) = 5$ by using Galerkin method.	**8**	**CO1**
Q7	According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is $30^\circ C$ and the substance cools from $100^\circ C$ to $70^\circ C$ in 15 minutes. Determine when the temperature will be $40^\circ C$.	**8**	**CO4**
Q8	Solve $u_t = 5u_{xx}$ with $u(0,t) = 0;\ u(5,t) = 40$ and $u(x,0) = \begin{cases} 20x & for\ 0 < x \le 2 \\ 40 & for\ 2 < x \le 5 \end{cases}$ for five time steps having $h = 1$ by using Schmidt method.	**8**	**CO2**

Q9	A point $p(7,3,1)^T$ is attached to a frame F_{noa} and is subjected to the following transformations. Determine the coordinates of the point relative to the reference frame at the conclusion of transformation. **1.** Rotation of $90°$ about the ***z***-axis, **2.** Followed by a rotation of $90°$ about the ***y***-axis, **3.** Followed by a translation of [4,-3,7].	**8**	**CO5**
Q10	Determine the value of y for $x=0.1$ and $x=0.2$ for $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ given that $y(0)=1$ by Runge-Kutta method of fourth order.	**8**	**CO1**
	OR		
Q10	Determine the value of y for $x=0.2$ and $x=0.4$ for $\frac{dy}{dx}=x+\left\|\sqrt{y}\right\|$ given that $y(0)=1$ by Euler's modified method with step size $h=0.2$.	**8**	**CO1**
SECTION-C			
Q11(A)	Solve $u_{xx}+u_{yy}=0$, over the square mesh of side four units satisfying the following boundary conditions: (i) $u(0,y)=0$ for $0\le y\le 4$ (ii) $u(4,y)=12+y$ for $0\le y\le 4$ (iii) $u(x,0)=3x$ for $0\le x\le 4$ (iv) $u(x,4)=x^2$ for $0\le x\le 4$.	**10**	**CO2**
Q11(B)	A point moves in a straight line towards a center of force $\frac{\mu}{(\text{distance})^3}$, starting from rest at a distance 'b' from the center of force. Show that the time of reaching a point distant 'c' from the center of force is $\frac{b}{\sqrt{\mu}}\sqrt{b^2-c^2}$ and its velocity then is $\frac{\sqrt{\mu}}{bc}\sqrt{b^2-c^2}$.	**10**	**CO4**
Q12(A)	Determine the nature of the critical point $(0,0)$ of the non-linear autonomous system $\frac{dx}{dt}=-x+2x^2+y^2, \frac{dy}{dt}=xy-y$ and also determine the stability of $(0,0)$ by Liapunov's direct method.	**10**	**CO3**
	OR		
Q12(A)	Consider the linear autonomous system $\frac{dx}{dt}=x+y, \frac{dy}{dt}=3x-y$ **(i)** Determine the nature of the critical point $(0,0)$ **(ii)** Determine the general solution of this system, and **(iii)** Determine the stability of $(0,0)$.	**10**	**CO3**
Q12(B)	For the following frame ***F***, determine the values of the missing elements and	**10**	**CO5**

	complete the matrix representation of the frame $F = \begin{bmatrix} ? & 0 & ? & 5 \\ 0.71 & ? & ? & 3 \\ ? & ? & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.		
	OR		
Q12(B)	A frame ***F*** was rotated about the ***x***-axis $90°$, then it was translated about the current ***a***-axis 3 inches before it was rotated about the ***z***-axis $90°$. Finally, it was translated about the current ***o***-axis 5 inches, then **(a)** Write an equation that describes the motions, and **(b)** Determine the final location of a point $p(1,5,4)^T$ attached to the frame relative to the reference frame.	**10**	**CO5**

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SECTION A

S. No.		Marks	CO
Q1	Classify the following partial differential equations $(i)\ u_{xx} - u_{xy} - 4u_{yy} + 3u_y = 0$ $(ii)\ u_{xx} + 6u_{xy} - u_{yy} - 17u_x = 0.$	4	CO2
Q2	Determine the value of y at $x = 0.1$ by Picard's method for only one approximation, given that $\frac{dy}{dx} = \frac{2y - x}{y + x}$, $y(0) = 1$.	4	CO1
Q3	Define center and spiral critical points of a linear autonomous system with examples.	4	CO3
Q4	Determine the nature of the critical point $(0,0)$ of the system $\frac{dx}{dt} = x + y, \frac{dy}{dt} = x - 2y$ and determine whether or not the critical point is stable.	4	CO3
Q5	A frame ***F*** has been moved 5 units along the z-axis and 10 units along the **x-**axis of the reference frame. Determine the new location of the frame, where $F = \begin{bmatrix} 0.7 & -0.7 & 0.5 & 10 \\ 0.3 & 0.2 & 0.4 & 2 \\ -0.7 & 0 & 0.6 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.	4	CO5

SECTION B

S. No.		Marks	CO
Q6	Solve the differential equation $\frac{d^2y}{dx^2} - 2y = x,\ x \in [0,1]$ with the boundary conditions $y(0) = 0, \& \ y(1) = 1$ by using method of least square.	8	CO1
Q7	A boat is rowed with a velocity ***u*** across a stream of width ***d***. If the velocity of the current is directly proportional to the product of the distances from the two banks, determine the equation of the path of the boat and the distance down the stream to the point, where it lands.	8	CO4
Q8	Solve $u_t = u_{xx}$ subject to the conditions $u(x,0) = 0;\ u(0,t) = 0$ and $u(1,t) = 10t$. Evaluate u for $t = \frac{1}{8}$ in two steps, using Crank- Nicholson's scheme.	8	CO2
Q9	A point $p(4,2,1)^T$ is attached to a frame F_{noa} and is subjected to the following	8	CO5

	transformations. Determine the coordinates of the point relative to the reference frame at the conclusion of transformation. **1.** Rotation of $90°$ about the ***y***-axis, **2.** Followed by a translation of [4,-3,7], and **3.** Followed by a rotation of $90°$ about the ***z***-axis.		
Q10	Determine the value of y for $x=0.1$ and $x=0.2$ for $\frac{dy}{dx}=\frac{y^2+x}{y^2+2x}$ given that $y(0)=1$ by Runge-Kutta method of fourth order.	**8**	**CO1**
	OR		
Q10	Determine the value of y for $x=0.2$ and $x=0.4$ for $\frac{dy}{dx}=xy$ given that $y(0)=1$ by Euler's modified method with step size $h=0.2$.	**8**	**CO1**
SECTION-C			
Q11(A)	Solve $u_{tt}=4u_{xx}$ upto $t=0.5$ with spacing $h=1$ given that $u(x,0)=x(4-x)$, $u(0,t)=0=u(4,t)$; $u_t(x,0)=0$.	**10**	**CO2**
Q11(B)	A particle is performing a simple harmonic motion of period ***T*** about a center *O* and it passes through a point *P*, where $OP=b$ with velocity ***v*** in the direction *OP*. Prove that the time which elapses before it returns to *P* is $\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$.	**10**	**CO4**
Q12(A)	Determine the nature of the critical point $(0,0)$ of the non-linear autonomous system $\frac{dx}{dt}=x-x^2+4y, \frac{dy}{dt}=6x-y+2xy$ and also determine the stability of $(0,0)$ by Liapunov's direct method.	**10**	**CO3**
	OR		
Q12(A)	Consider the linear autonomous system $\frac{dx}{dt}=x+3y, \frac{dy}{dt}=3x+y$ **(i)** Determine the nature of the critical point $(0,0)$ **(ii)** Determine the general solution of this system, and **(iii)** Determine the stability of $(0,0)$.	**10**	**CO3**
Q12(B)	For the following frame ***F***, determine the values of the missing elements and complete the matrix representation of the frame $F=\begin{bmatrix} ? & 0 & ? & 3 \\ 0.5 & ? & ? & 9 \\ 0 & ? & ? & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.	**10**	**CO5**
	OR		
Q12(B)	A frame ***F*** was rotated about the ***y***-axis $90°$, followed by a rotation about the ***o***-axis $30°$, followed by a translation of 5 units along the ***n***-axis, and finally, a translation of	**10**	**CO5**

	4 units along the ***x***-axis $90°$, then **(a)** Write an equation that describes the motions. **(b)** Determine the total transformation matrix. **(c)** Determine the final location of a point $p(1,1,1)^T$ attached to the frame relative to the reference frame.		