Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2018

Course: Mathematical Methods **Semester: I**
Course Code: DSQT 1003

Programme: BA (H) Energy Economics
Time: 03 hrs. **Max. Marks: 100**

Instructions: Answer **all** the questions from Section A, **Four** questions from Section B**, Three** questions from Section C and Section D is **compulsory.**

SECTION A (5*4 = 20 marks)

S. No.	Find the derivative *dy/dx* of the following functions (Q 1 to Q 3)	**Marks**	**CO**
Q 1	$y = (5x^2 + 3)^3$	**4**	**1**
Q 2	$y = (3x^2 - 2)(x + 1)$	**4**	**1**
Q 3	$y = (3x^4 - 1)/(2x^3 + 5)$	**4**	**1**
	Find the integration of the following functions (Q 4 and Q 5)	**4**	
Q 4	$y = \int (2x^3 - x^2)\, dx$	**4**	**1**
Q 5	$y = \int_0^5 (4x^2 + 6x + 3)dx$	**4**	**1**

SECTION B (4*5 = 20 marks)

Q 1	Determine the rank (ρ) of the following matrix. $B = \begin{bmatrix} 12 & 0 & 3 \\ 9 & 2 & 5 \\ 4 & 6 & 1 \end{bmatrix}$	**5**	**1**
Q 2	Use implicit differentiation to find the derivative *dy/dx* for the following equation. $7x^4 + 3x^3y + 9xy^2 = 62$	**5**	**1**
Q 3	Check whether the following function is concave or convex at $x = 2$ $y = (5x^2 - 4)^2$	**5**	**2**
Q 4	Find the critical value(s) at which the following function is optimized. $y = x^3 - 18x^2 + 96x - 80$ Determine if the function is at relative maximum or minimum at the critical value(s).	**5**	**3**
Q 5	Assume that the rate of net investment is given as $I = 10t^{3/5}$, and capital stock (K) at $t = 0$ is 750. Find the capital stock function K.	**10**	**3**

SECTION-C (3*10 = 30 marks)			
Q 1	Assume that the marginal cost (MC) is given as $MR = 24 + 4Q - 12Q^2$, and fixed cost (*FC*) is 45. Find total cost (*TC*), average cost (*AC*) and variable cost (*VC*) functions.	**10**	**4**
Q 2	The total cost function is given as $C(x) = x^3 - 5x^2 + 60x$, $x \geq 0$, where x represents units of output. (a) Compute the marginal cost function $C'(x)$. (b) Find the value of *x* at which average cost (AC) is minimum.	**10**	**4**
Q 3	Assume that the total revenue function is $R = 1400Q - 6Q^2$and total cost function is $C = 1500 + 8Q$, and $Q > 0$. (a) Find the level of output at which profit is maximum. (b) Calculate the maximum profit.	**10**	**3**
Q 4	Let B is a 3x3 matrix given as B = $\begin{bmatrix} 14 & 0 & 6 \\ 9 & 5 & 0 \\ 0 & 11 & 8 \end{bmatrix}$. Compute the inverse of matrix B.	**10**	**1**
SECTION-D (2*15 = 30 marks)			
Q 1	Use Lagrange multiplier to optimize the following function: $z = 4x^2 + 3xy + 6y^2$ subject to the constraint $x + y = 56$	**15**	**3**
Q 2	Use Cramer's rule to solve for the unknowns in the following system of equations. $11x - y - z = 31$ $-x + 6y - 2z = 26$ $-x - 2y + 7z = 24$	**15**	**2**