Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2019

Programme Name:	**B. Tech. ADE**	**Semester : VI**
Course Name :	**Applied Numerical Techniques**	**Time : 03 hrs**
Course Code :	**MATH 305**	**Max. Marks : 100**
Nos. of page(s) :	**03**	

Instructions: Attempt all questions from **Section A** (each carrying 5 marks); attempt all questions from **Section B** (each carrying 8 marks)**;** attempt the question from **Section C** (each carrying 20 marks)**. Scientific calculator is allowed.**

SECTION A
(Attempt all questions)

<table>
<tr><th>S. No.</th><th></th><th>Marks</th><th>CO</th></tr>
<tr><td>Q 1</td><td>Consider the following boundary value problem (BVP).
$\frac{d^2y}{dx^2} - y = x^4, 0 \le x \le 1,$
with the boundary conditions $y(0)=0$ and $y(1)=0$. Choose two basis functions $\phi_1(x)$ and $\phi_2(x)$ for an approximate solution $ý = a_1\phi_1(x) + a_2\phi_2(x)$. Hence find the residual.</td><td>5</td><td>CO6</td></tr>
<tr><td>Q 2</td><td>Use two approximations of Picard's method to obtain y for $x=0.2$. Given:
$\frac{dy}{dx} = x - y$ with $y(0)=1$.</td><td>5</td><td>CO5</td></tr>
<tr><td>Q 3</td><td>By considering three terms of Taylor's series, evaluate $y(1.1)$ from the following differential equation:
$\frac{dy}{dx} = x + y$ with $y(1)=0$.</td><td>5</td><td>CO5</td></tr>
<tr><td>Q 4</td><td>Evaluate $I = \pi \int_0^1 y^2 dx$ using Simpson's rule:<table><tr><td>x:</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>y:</td><td>1</td><td>0.9896</td><td>0.9589</td><td>0.9089</td><td>0.8415</td></tr></table></td><td>5</td><td>CO2</td></tr>
</table>

SECTION B
(Q5, Q6, Q7 are compulsory and Q8, Q9 have internal choices)

<table>
<tr><td>Q 5</td><td>Given that:
$\frac{dy}{dx} = \log_{10}(x+y)$ with $y(0)=1$
Find y for $x=0.2$ using Euler's modified method correct upto four decimal places (take $h=0.2$¿.</td><td>8</td><td>CO5</td></tr>
</table>

<table>
<tr><td>Q 6</td><td>Using Lagrange’s interpolation, evaluate $\left[\frac{df}{dx}\right]_{at\,x=4}$ from the following data:
<table><tr><td>x:</td><td>0</td><td>2</td><td>5</td><td>1</td></tr><tr><td>$f(x)$</td><td>0</td><td>8</td><td>125</td><td>1</td></tr></table></td><td>8</td><td>CO2</td></tr>
<tr><td>Q 7</td><td>Solve the equation $\frac{dy}{dx}=x+y$ with initial condition $y(0)=1$ by Runge-Kutta method of order four, from $x=0$ to $x=0.2$ with step length $h=0.1$.</td><td>8</td><td>CO5</td></tr>
<tr><td>Q8</td><td>Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ in $0<x<5$, $t\geq 0$ given that $u(x,0)=20$, $u(0,t)=0$, $u(5,t)=100$.
Compute u for one time step with $h=1$ by Crank-Nicolson method.
OR
Using Taylor’s series, find the solution of the differential equation $x\frac{dy}{dx}=x-y$ with $y(2)=2$ at $x=2.1$ correct to five decimal places.</td><td>8</td><td>CO5</td></tr>
<tr><td>Q9</td><td>Find the positive root of $x^4-x=10$ correct to three decimal places using Newton-Raphson method.
OR
The graph of $y=2\sin x$ and $y=\log x+c$ touch each other in the neighborhood of point $x=8$. Find c and the coordinates of point of contact.</td><td>8</td><td>CO3</td></tr>
<tr><td colspan="4">SECTION-C
(Q 10A, Q10B are compulsory and Q11A and Q11B have internal choices)</td></tr>
<tr><td>Q 10 A</td><td>Using Milne’s method, solve $\frac{dy}{dx}=1+y^2$ with $y(0)=0$, $y(0.2)=0.2027$;
$y(0.4)=0.4228$; $y(0.6)=0.6841$, obtain $y(0.8)$,</td><td>10</td><td>CO5</td></tr>
<tr><td>Q10B</td><td>Consider the following boundary value problem (BVP)
$\frac{d^2u}{dx^2}-u=x, 0\leq x\leq 1$
with $u(0)=0, u(1)=0$.
Find an approximate solution $\acute{u}(x)=a_1\phi_1(x)+a_2\phi_2(x)$ by Galerkin’s method. Consider the basis functions $\phi_1(x)=x(x-1)$ and $\phi_2(x)=x^2(x-1)$.</td><td>10</td><td>CO6</td></tr>
<tr><td>Q11A</td><td>Given that :
$\sqrt{12500}=111.803399$, $\sqrt{12510}=111.848111$, $\sqrt{12520}=111.892806$
$\sqrt{12530}=111.937483$.
Using Gauss’s Backward formula, evaluate $\sqrt{12516}$.</td><td>10</td><td>CO1</td></tr>
</table>

<table>
<tr>
<td></td>
<td>OR

By means of Newton’s divided difference formula, find the values of $f(8)$ and $f(15)$ from the following table:
<table>
<tr><td>x:</td><td>4</td><td>5</td><td>7</td><td>10</td><td>11</td><td>13</td></tr>
<tr><td>$f(x)$</td><td>48</td><td>100</td><td>294</td><td>900</td><td>1210</td><td>2028</td></tr>
</table>
</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q11B</td>
<td>Solve equations $27x+6y-z=85$; $x+y+54z=110$; $6x+15y+2z=72$ using Gauss-Seidel method. Use only four iterations.

OR

Apply Doolittle method of LU decomposition to solve the equations:

$3x+2y+7z=4$; $2x+3y+z=5$; $3x+4y+z=7$</td>
<td>10</td>
<td>CO4</td>
</tr>
</table>

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Course Name : Applied Numerical Techniques
Course Code : MATH 305
Nos. of page(s) : 03

Semester : VI
Time : 03 hrs
Max. Marks : 100

Instructions: Attempt all questions from **Section A** (each carrying 5 marks); attempt all questions from **Section B** (each carrying 8 marks)**;** attempt the question from **Section C** (each carrying 20 marks)**. Scientific calculator is allowed.**

SECTION A
(Attempt all questions)

S. No.		**Marks**	**CO**
Q 1	The Poisson equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-1$ defined in the domain D where $D=\{(x,y),-1\le x,y\le 1\}$ with $u=0$ on $x=\pm 1$ and $y=\pm 1$. Using the trial function $\phi(x)=(1-x^2)(1-y^2)$ for an approximate solution $\acute{u}=a\phi(x)$. Hence find the residual.	**5**	**CO6**
Q 2	Use single approximations of Picard's method to obtain y for $x=0.1$. Given: $$\frac{dy}{dx}=3x+y^2 \text{ with } y(0)=1.$$	**5**	**CO5**
Q 3	Given $\frac{dy}{dx}=\frac{y-x}{y+x}$ with $y(0)=1$. Find y approximately for $x=0.04$ taking step size $h=0.02$ by Euler's method.	**5**	**CO5**
Q 4	Evaluate $I=\int_0^1 \frac{1}{1+x^2}dx$ using Simpson's one third rule taking $h=\frac{1}{4}$.	**5**	**CO2**

SECTION B
(Q5, Q6, Q7 are compulsory and Q8, Q9 have internal choices)

Q 5	Find $y(1)$ for $\frac{dy}{dx}=2y+3e^x$ with $y(0)=0$ using Taylor's series method up to fifth derivative. Compare it with the exact solution.	**8**	**CO5**
Q 6	Using Newton forward interpolation, find $\frac{dy}{dx}$ at $x=0.1$ from the following table: x: 0.1, 0.2, 0.3, 0.4 y: 0.9975, 0.9900, 0.9776, 0.9604	**8**	**CO2**
Q 7	Using Milne's method, solve $\frac{dy}{dx}=\frac{1}{2}(x+y)$ with $y(0)=2$, $y(0.5)=2.636$; $y(1)=3.595$;	**8**	**CO5**

	$y(1.5)=4.968$, find $y(2)$.		
Q8	Using Euler's modified method, obtain $y(0.2)$ from the following differential equation $\frac{dy}{dx}=x+\lvert\sqrt{y}\rvert$ with initial condition $y(0)=1$. (take $h=0.2$¿ **OR** Using Runge-Kutta method of fourth order, solve for y at $x=1.2$ from $\frac{dy}{dx}=\frac{2xy+e^x}{x^2+xe^x}$ given $y(1)=0$ (take $h=0.2$).	**8**	**CO5**
Q9	Consider the graph of $\cos x$ for the non-negative values of $x\in R$ (set of real numbers). The oblique line $y=x$ cuts this graph of at the point $P(x,y)$.Use bisection method to obtain the abscissa of point P correct to 3 decimal places. **OR** Compute root of the equation $x^2e^{-x/2}=1$ in the interval $[0,2]$ using secant method. The root should be correct to three decimal places.	**8**	**CO3**
SECTION-C **(Q 10A, Q10B are compulsory and Q11A and Q11B have internal choices)**			
Q 10 A	Using Crank- Nicolson method, solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ in $0<x<5$, $t\geq 0$ given that $u(x,0)=20$, $u(0,t)=0$, $u(5,t)=100$. Compute u for one time step with $h=1$.	**10**	**CO5**
Q10B	Using two parameters, solve the following boundary value problem by Galerkin's method. $\frac{d^2u}{dx^2}+u=1+x^2;\ u(0)=u(1)=0$	**10**	**CO6**
Q11A	If $f(x)$ is a polynomial of degree four and given that: $f(4)=270, f(5)=648, \Delta f(5)=682, \Delta^3 f(4)=132$ Find $f(6)$ and $f(7)$ and hence find the value of $f(5.8)$ using Gauss's backward formula. **OR** A curve $y=f(x)$ passes through the points $(0,18),(1,10),(3,-18)$ and $(6,90)$. Find the slope of the curve at $x=2$ by using Newton's divided difference interpolation formula.	**10**	**CO1**

Q11B	Solve the equations $x+y+z=3$; $x-y+z=4$; $x+y-z=5$ by Choleski decomposition method. **OR** Solve the following system of equations by Gauss-Seidel method correct to three decimal places: $27x+6y-z=85$; $x+y+54z=110$; $6x+15y+2z=72$	**10**	**CO4**