

Name:	
Enrolment No:	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2019

Course: Mathematics-I (MATH-1002)

Programme: All SOCS Branches

No. of pages: 2

Instructions: All sections are compulsory

Semester: I

Time: 03 hrs.

Max. Marks: 100

SECTION A
Attempt all Questions

S. No.	Question	Marks	CO
Q 1	Check whether the given statement $(p \wedge q) \vee (q \wedge \sim r)$ is tautology or not ?	4	CO2
Q2	Let $\lambda_1 = 3$ and $\lambda_2 = 5$ be two the eigen values of the matrix A of order 3×3 and $Trace(A) = 12$. Find the determinant of inverse of the matrix A .	4	CO1
Q3	Let the group $G = \{1, 2, 4, 5, 7, 8\}$ under multiplication modulo 9 be a cyclic. Find its all generator.	4	CO5
Q4	Find the n^{th} derivative of the function $y = e^{3x}(2x + 3)^3$	4	CO3
Q5	Evaluate $\int_0^2 \int_1^{e^x} dy dx$	4	CO4

SECTION B
Attempt all Questions

Q6	Test the given vectors are $X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$, $X_4 = (-3, 7, 2)$ linearly dependence or not? If yes, find the relation between them.	10	CO1
Q7	Check whether the two statements $p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$ are equivalence?	10	CO2
Q8	Change the order of integration and evaluate: $\int_0^1 \int_y^{\sqrt{2-y^2}} \frac{y}{\sqrt{x^2+y^2}} dx dy$.	10	CO3
Q9	Define subgroup and let set $H = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}, a \neq 0, a, b \in \mathbb{R} \right\}$ be a subset of the multiplicative group G of 2×2 non-singular matrices over \mathbb{R} . Is the given set H a subgroup of G ? <p style="text-align: center;">OR</p> Define the order of an element of group and also find the order of each element of an abelian group $G = \{1, 3, 5, 7\}$ under multiplication mod 8.	10	CO5

SECTION-C
Attempt all Questions

Q10(A)	Let \mathbb{R} is the set of integers mod 6 under addition and multiplication mod 6. Show that \mathbb{R} is a ring with unit element.	10	CO5
Q10(B)	If \mathbb{R} is the additive group of real numbers and \mathbb{R}_+ the multiplicative group of positive real numbers, prove that the mapping $f: \mathbb{R} \rightarrow \mathbb{R}_+$ defined by $f(x) = e^x \forall x \in \mathbb{R}$ is an isomorphism of \mathbb{R} onto \mathbb{R}_+ .	10	CO5
Q11(A)	<p>If $u = xyz, v = xy + yz + zx, w = x + y + z$, then compute Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.</p> <p style="text-align: center;">OR</p> <p>A rectangular box open at the top is to have volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction.</p>	10	CO3
Q11(B)	<p>Evaluate $\iint_R x^2 dx dy$, where R is the region in the first quadrant bounded by $xy = 16, x = y, y = 0$ and $x = 8$.</p> <p style="text-align: center;">OR</p> <p>Evaluate $\iiint xyz(x^2 + y^2 + z^2) dx dy dz$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$</p>	10	CO4