Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2019

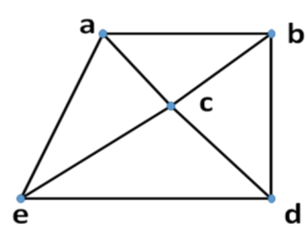
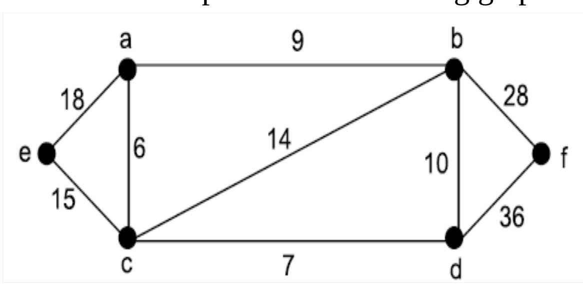
Course: Discrete Mathematical Structures **Semester: III (2019-2020)**
Course Code: CSEG 2006 **Time: 03 hrs.**
Programme: B.Tech (All SoCS Branches) **Max. Marks: 100**

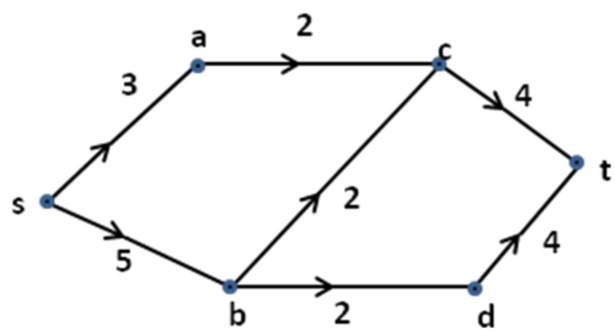
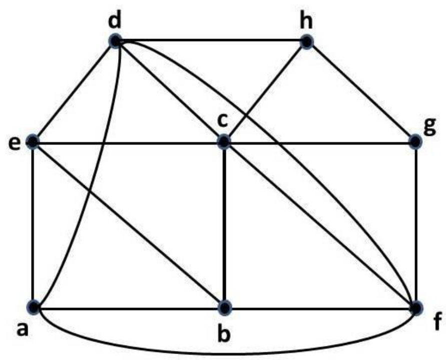
Instructions: Attempt all questions from **Section A** (each carrying 4 marks); all questions from **Section B** (each carrying 10 marks) and all questions from **Section C** (carrying 20 marks).

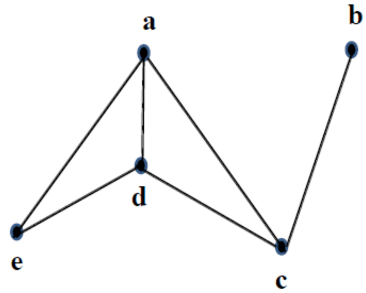
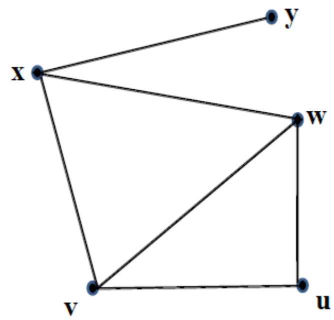
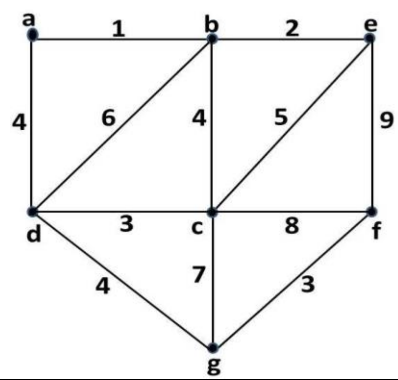
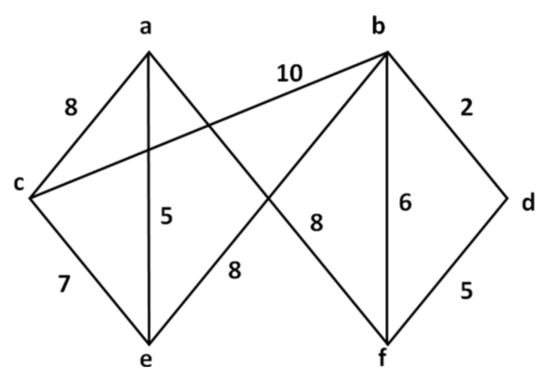
SECTION A

S. No.		Marks	CO
Q1	Check whether the set of all integers $\mathbb{Z}$ is a countable infinite set or not. Give the explanation.	**4**	**CO1**
Q2	Check whether the set of vectors $\{(1,2,3),(1,-1,-1),(3,2,1),(2,1,-1)\}$ is basis or not of $\mathbb{R}^3$.	**4**	**CO2**
Q3	G is a non-directed simple graph with 12 edges. If G has 6 vertices each of 3 degrees and the rest have the degree less than 3, determine the minimum number of vertices G can have.	**4**	**CO3**
Q4	A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many vertices of degree 1 does it have?	**4**	**CO4**
Q5	Check whether the transformation $T:\mathbb{R}^3(\mathbb{R})\to\mathbb{R}^3(\mathbb{R})$ defined as $T(x,y,z)=(x,y,kz),\ k\in\mathbb{R}$ is a linear transformation or not.	**4**	**CO2**

SECTION B

Q6	Using the decomposition theorem, determine the chromatic polynomial and hence the chromatic number of the graph as shown below.  	**10**	**CO3**
Q7	Using Dijkstra's algorithm, determine the length of the shortest path between the vertices **e** to **f** and hence the shortest path in the following graph.  	**10**	**CO3**

Q8	Give an example of each graph which is **(i)** Cycle (C_5) **(ii)** Eulerian **(iii)** Hamiltonian, **(iv)** Eulerian but non-Hamiltonian and **(v)** Hamiltonian but non- Eulerian.	**10**	**CO3**
Q9	Determine the solution of the following non-homogeneous recurrence relation $y_n + 4y_{n-1} + 4y_{n-2} = n^2 - 3n + 5$, with $y_0 = 0$, $y_1 = 1$.	**10**	**CO1**
	OR		
Q9	Using the generating function method, determine the solution of the recurrence relation $y_{n+2} - 2\,y_{n+1} + y_n = 2^n$ with the conditions $y_0 = 2$ and $y_1 = 1$.	**10**	**CO1**
SECTION-C			
Q10(A)	Show that the set (V) of all $m \times n$ matrices with elements as real numbers is a vector space over the field $(\mathbb{R}, +, \bullet)$ with "addition of matrices" is the internal composition and "multiplication of a matrix by a scalar" is the external composition in V.	**10**	**CO2**
Q10(B)	Determine the maximum flow for the network as shown below using Ford-Fulkerson algorithm. Determine the cut with capacity equal to this maximum flow. 	**10**	**CO4**
Q11(A)	Define graph vertex colouring. Explain Welch-Powell algorithm and using this algorithm determine the coloring of the graph as shown below and hence find the chromatic number $\chi(G)$. 	**10**	**CO3**
	OR		

Q11(A)	Define isomorphism between two graphs. Determine whether the given pair of graphs is isomorphic or not. Give the explanation.  **(G)**  **(H)**	**10**	**CO3**
Q11(B)	Using Prim's algorithm, determine a minimal spanning tree from the weighted graph as given below  	**10**	**CO4**
	OR		
Q11(B)	Using Kruskal's algorithm, determine a minimal spanning tree of the weighted graph given below.  	**10**	**CO4**