Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

Supplementary Examination, May 2020

Course: Ring Theory & Linear Algebra-I **Semester: IV**
Program: B.Sc Mathematics (Hons.) **Time 3 hrs.**
Course Code: MATH2031 **Max. Marks: 100**
Nos. of page(s) : 1

Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

PART A

Instructions: PART A contains 20 questions for a total of 60 marks. It contains 10 multiple choice questions, 8 multiple answer questions and 3 True/False questions. Multiple answer questions may have more than one correct option. Select all the correct options. You need to answer PART A within the slot from 2:00 PM to 6:00 PM on 10th July 2020. The due time for PART A is 5:00 PM on 10th July 2020. After the due time, the PART A will not be available.

S. No.		Marks	CO
Q1 (i)	Let F be a field and let T be the linear operator on F^2 defined by $T(x,y) = (x+y,x)$. C the correct options: A. T is singular B. T is non-singular C. T is invertible D. None of these	**3**	**CO4**
Q1 (ii)	Let F be a field and let T be the linear operator on F^2 defined by $T(x,y) = (x+y,x)$. the correct options: A. $TT^{-1} = I$ B. $T^{-1}(x,y) = (y, x-y)$ C. $T^{-1}T = I$ D. None of these	**3**	**CO4**

Q1 (iii)	Let T is a linear transformation from V into W and dimension of V = dimension o Choose correct options A. If T is invertible, then T is onto B. If T is onto, then T is invertible C. If T is invertible, then T is one-one D. If T is one-one, then T is onto	**3**	**CO4**
Q1 (iv)	Let V be a vector space and T a linear transformation from V into V. Let the interse the range of T and the null space of T is the zero subspace of V. If $T(T\alpha) = 0$, Then A. $T\alpha = V$ B. $T(T\alpha) = 0, \forall\, \alpha \in V$ C. $T\alpha = 0$ D. None of these	**3**	**CO4**
Q1 (v)	Choose correct options for a vector space V of dimension n: A. The rank of the zero transformation is 0 B. the rank of the identity transformation is n. C. The nullity of the zero transformation is n D. the nullity of the identity transformation is 0.	**3**	**CO4**

Q1 (vi)	Let F be a subfield of the field of complex numbers. Let $$\begin{aligned}\alpha_1 &= (1,2,0,3,0)\\ \alpha_2 &= (0,0,1,4,0)\\ \alpha_3 &= (0,0,0,0,1)\end{aligned}$$ Let W be a subspace of F^5 spanned by $\alpha_1, \alpha_2, \alpha_3$. Which vector is in A. $(-3,-5,1,-5,2)$ B. $(2,4,6,7,8)$ C. $(-3,-6,1,-5,2)$ D. none of these	**3**	**CO3**
Q1 (vii)	Let F be a subfield of the field of complex numbers. In F^3 the vect $$\begin{aligned}\alpha_1 &= (3,0,-3)\\ \alpha_2 &= (-1,1,2)\\ \alpha_3 &= (4,2,-2)\end{aligned}$$ are: A. linearly independent B. linearly dependent C. are standard basis for F^3 D. none of these	**3**	**CO3**

Q1 (viii)	The dimension of the space of $n \times n$ matrices over the field F is A. n B. $2n$ C. n^2 D. none of these	**3**	**CO3**
Q1 (ix)	Let V be the vector space of all 2×2 matrices over the field F. Let W be the subspace o the form, $\begin{bmatrix} x & -x \\ y & z \end{bmatrix}$. The dimension of W is: A. 1 B. 2 C. 3 D. 4	**3**	**CO3**
Q1 (x)	Let $\alpha_1 = (1,2)$ and $\alpha_2 = (3,4)$ forms a basis for R^2. Consider a linear transformat $R^2 \longrightarrow R^3$, such that $$\begin{aligned} T\alpha_1 &= (3,2,1) \\ T\alpha_2 &= (6,5,4) \end{aligned}$$ Then the image of $T(1,0)$ is A. (1,2,3) B. (1,3,2) C. (0,2,1) D. (0,1,2)	**3**	**CO4**

Q1 (xi)	Choose a linear transformation from $T : R^2 \longrightarrow R^2$ A. $T(x, y) = (1 + x, y)$ B. $T(x, y) = (y, x)$ C. $T(x, y) = (x^2, y)$ D. $T(x, y) = (\sin x, y)$	**3**	**CO3**
Q1 (xii)	Let V is any vector space over a field F. Choose the correct options. A. the subset consisting of the zero vector alone is a subspace of B. the space V is a subspace of V C. any subset of V is a subspace of V D. none of these	**3**	**CO3**
Q1 (xiii)	Let V be the set of all pairs (x, y) of real numbers, and let F be the field of real numbers $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ $c(x, y) = (cx, y)$ With these operations, V is a vector space over the field of real numbers. True or False	**3**	**CO3**
Q1 (xiv)	Let V be the set of all pairs (x, y) of real numbers, and let F be the field of real numbers $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, 0)$ $c(x, y) = (cx, 0)$ With these operations, V is a vector space over the field of real numbers. True or False	**3**	**CO3**

Q1 (xv)	A ring homomorphism ϕ from a ring R to a ring S is a mapping from R to S that p some operations; for all $a,\ b$ in R, which of the operations are preserved. A. $\phi(a+b) = \phi(a) + \phi(b)$ B. $\phi(ab) = \phi(a)\phi(b)$ C. $\phi(a) = \phi(b)$ D. none of these	**3**	**CO2**
Q1 (xvi)	Let R be a commutative ring of characteristic 2. Choose correct optio A. $2x = 0 \,\forall\, x \in R$ B. the mapping $a \longrightarrow a$ is a ring homomorphism from R to R C. the mapping $a \longrightarrow a^2$ is a ring homomorphism from R to R D. none of these	**3**	**CO2**
Q1 (xvii)	Let $R[x]$ denote the set of all polynomials with real coefficients and let A denote the s all polynomials with constant term 0. Then A is an ideal of R[x] and A is equal to: A. $< x >$ B. $< x^2 >$ C. $< 1 >$ D. none of these	**3**	**CO1**

Q1 (xviii)	In the ring of integers Z, the ideal 5Z is A. not a subring of Z B. a group with respect to multiplication C. a prime ideal D. none of these	**3**	**CO1**
Q1 (xix)	Choose all the true statements about the integral domain. A. An integral domain is a commutative ring with unity and no zero di B. The ring of integers is an integral domain. C. A finite integral domain is a field. D. The characteristic of an integral domain is 0 or prime.	**3**	**CO1**
Q1 (xx)	Consider the following two statements. i. $A = \{0, 2, 4\}$ is a subring of the ring Z_6, the integers modulo 6 ii. 4 is the unity in the subring A. Choose the correct option. A. Only (i) is true B. Only (ii) is true C. Both are true D. Both are false	**3**	**CO1**

PART B			
The link for PART B will be available from 2:00 PM on 10th July 2020 to 2:00 PM on 11th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BRANCH NAME_ROLL NUMBER (for example: 500077624_CCVT_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.			
Q 1	Show that $Q[\sqrt{2}] = \{a + b\sqrt{2}: a, b \in Q\}$ is a field.	**6**	**CO1**
Q 2	Let $R[x]$ denote ring of all polynomials with real coefficients. Show that the mapping $f(x) \rightarrow f(1)$ is a ring homomorphism from $R[x]$ to R. In addition, find the kernel of the homomorphism.	**6**	**CO2**
Q 3	Let V be the set of all pairs (x, y) of real numbers, and let F be the field of real numbers. Define $$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$ $$c(x, y) = (cx, y)$$ Is V, with these operations, a vector space over the field of real numbers?	**4**	**CO3**
Q 4	Let R be a field of real numbers. Suppose $$\alpha_1 = (1,2,0,3,0), \alpha_2 = (0,0,1,4,0), \alpha_3 = (0,0,0,0,1)$$ Explain the subspace W of R^5 spanned by α_1, α_2 and α_3. Show that (-3,-6, 1, - 5, 2) is in W, whereas (2, 4, 6, 7, 8) is not.	**10**	**CO3**
Q 5	Let F be a field and let T be the linear operator on F^2 defined by $$T(x, y) = (x + y, x)$$ Show that T is non-singular and onto. In addition, find the inverse of T.	**10**	**CO4**
Q 6	Define rank and nullity of a linear transformation from a finite dimensional vector space V.	**4**	**CO4**