UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2020

Programme Name: B. Sc. (Hons.) Mathematics   Semester : V
Course Name : Mathematical Modelling and Graph Theory   Time : 03 hrs
Course Code: MATH 3024   Max. Marks : 100

	Section A (All questions are compulsory, each question is of 5 marks)	

1.	Alternative solution exist in a linear programming problem when
	  A. one of the constraint is redundant
	  B. objective function is parallel to one of the constraints
	  C. two constraints are parallel
	  D. all of the above
		CO4

2.	Radius of a graph G, denoted by *rad* (G) is defined by....?
	  A. max $\{e(v): v$ belongs to $V \}$
	  B. min $\{ e(v): v$ belongs to $V\}$
	  C. max $\{ d(u, v): u, v$ belongs to V, u does not equal to $v\}$
	  D. min $\{ d(u, v): u , v$ belongs to V, u does not equal to $v\}$
		CO6

3.	Given some initial seed, say x_0, we generate a sequence of random points by the rule
	$x_{n+1} = (a x_n + b) \, mod \, (c)$ then this methodology generates
	  A. Exactly c random points
	  B. Exactly $c - 1$ random points
	  C. Exactly $c + 1$ random points
	  D. More than $c + 1$ random points
		CO3

4.	Let $P_n(x)$ be the Legendre polynomial of degree $n \geq 0$. If $1 + x^{10} = \sum_{n=0}^{10} C_n P_n(x)$ then C_5 equals
	
	  A. 0
	  B. 2/11
	  C. 11/2
	  D. 1
		CO1

5.	If y is the solution of $y'' + 5 y' + 5 x = 1, y(0) = 0, y'(0) = 1$; then Laplace transform of y is
	
	  A. $\dfrac{s+1}{s(s^2 + 5s + 5)}$
	
	  B. $\dfrac{s^2 + 5s + 1}{s(s^2 + 5s + 5)}$
	
	  C. $\dfrac{1}{s(s^2 + 5s + 5)}$
	
	  D. $\dfrac{s}{s(s^2 + 5s + 5)}$
		CO2

6.	Which of the following is not one of the assumptions of an M/M/1 model? A. Arrivals are independent of preceding arrivals but the arrival rate does not change over time. B. Arrivals are served on a last-in, first-served basis. C. Service times follow the negative exponential probability distribution. D. Arrivals follow the Poisson distribution and come from an infinite population.	CO5

SECTION B
(All questions are compulsory and Q11 has internal choices, each question is of 10 marks)

7.	Consider a small harbour with unloading facilities for ships. Only one ship can be unloaded at any one time. Ships arrive for unloading of cargo at the harbor, and the time between the arrivals of successive ships varies from 15 to 145 min. The unloading time required for a ship depends on the type and amount of cargo and varies from 45 to 90 min. Answers the following questions: **1.** What are the average and maximum times per ship in the harbour? **2.** If the *waiting time* for a ship is the time between its arrival and the start of unloading, what are the average and maximum waiting times per ship? **3.** What percentage of the time are the unloading facilities idle? **4.** What is the length of the longest queue?	CO5
8.	Solve the second-order initial-value problem $\dfrac{d^2 y}{dt^2} + 2\dfrac{dy}{dt} + 2y = e^{-t}$, $y(0) = 0$ and $y'(0) = 0$ using the Laplace transform method.	CO2
9.	A farmer has 30 acres on which to grow tomatoes and corn. Each 100 bushels of tomatoes require 1000 gallons of water and 5 acres of land. Each 100 bushels of corn require 6000 gallons of water and 2.5 acres of land. Labour costs are $1 per bushel for both corn and tomatoes. The farmer has available 30,000 gallons of water and $750 in capital. He knows that he cannot sell more than 500 bushels of tomatoes or 475 bushels of corn. He estimates a profit of $2 on each bushel of tomatoes and $3 on each bushel of corn. **a.** How many bushels of each should he raise to maximize profits? **b.** Next, assume that the farmer has the opportunity to sign a nice contract with a grocery store to grow and deliver at least 300 bushels of tomatoes and at least 500 bushels of corn. Should the farmer sign the contract? Support your recommendation.	CO4
10.	Find a power series solution of the equation $y'' + 2x\,y' + y = 3e^x$	CO1
11.	Find the area trapped between the two curves $y = x^2$ and $y = 6 - x$ and the x and y axes using the simulation technique. **OR** Use the middle-square method to generate 10 random numbers using $x_0 = 1009$.	CO3

SECTION C
(Q12 is of 20 marks and it has internal choices)

12

The Adjency matrix of a graph is given by A, Plot the graph and determine whether it is
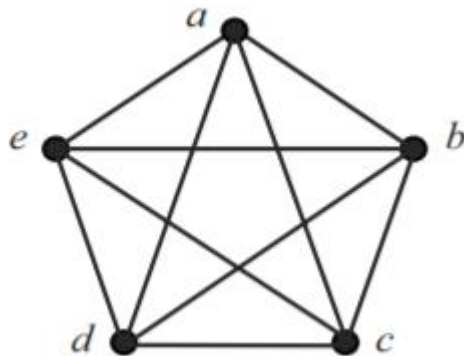A) Bipartite
B) Regular
C) Eulerian

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

and explain your answer in detail.

OR

Consider the following graph

(a) How many different paths have c as an end vertex?
(b) How many different paths avoid vertex c altogether?
(c) What is the maximum length of a circuit in this graph? Give an example of such a circuit

CO6