Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, January 2021

Program Name: B.Sc. Mathematics (Hons.) — **Semester : I**
Course Name : Calculus — **Time : 03 hrs**
Course Code : MATH 1030 — **Max. Marks : 100**
Nos. of page(s) : 02

SECTION A
(Attempt all questions; Each question carries 5 marks)

S. No.		**CO**
Q1.	The value of $\lim_{x\to 0+} x^x$ is A. Undefined B. 0 C. 1 D. ∞	**CO1**
Q2.	Consider the quadratic equation $5x^2 - 3xy + 2y^2 + 3x - 8y - 7 = 0$. Which type of conic section is it? A. Parabola B. Ellipse C. Hyperbola D. Not possible to identify.	**CO4**
Q3.	The area of the parallelogram $PQRS$, where $P = (1,1)$, $Q = (2,3), R = (5,4), S = (4,2)$ is A. 5 B. 4 C. 3 D. 2	**CO4**
Q4.	The area of the region enclosed by $y = x$ and $y = x^2 - x$ is A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. 1 D. $\frac{4}{3}$	**CO3**
Q5.	Let $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ be unit vectors in direction of x, y and z −axis respectively. Then $\hat{\imath} \times \hat{\jmath}$ is A. $\hat{0}$ B. $\hat{\imath}$ C. $\hat{\jmath}$ D. $\hat{k}$	**CO4**
Q6.	Consider the parametric equations: $x = t^2 + t,\ \ y = 2t - 1, t \in [-2,1]$. Identify the curve. A. It is an equation of circle. B. It is an equation of straight line. C. It is an equation of parabola. D. It is an equation of hyperbola.	**CO4**

SECTION B (Q7-Q10 are compulsory and Q11 has internal choice; Each question carries 10 marks)		
Q7.	Find the volume of the solid generated by revolving the region bounded by the parabola $x = y^2 + 1, y = 0$ and the line $x = 3$ about the line $x = 3$.	**CO3**
Q8.	Represent the velocity $\vec{v}$ and acceleration $\vec{a}$ of the motion $\vec{r}(t) = (e^t \cos t)\hat{\imath} + (e^t \sin t)\hat{\jmath} + \sqrt{2}e^t\hat{k}$, in form of $\vec{v} = v_T\vec{T} + v_N\vec{N}$ and $\vec{a} = a_T\vec{T} + a_N\vec{N}$, where $\vec{T}$ and $\vec{N}$ are unit tangent vector and unit normal vector respectively.	**CO5**
Q9.	If $\sin^{-1} y = 2\ln(x+1)$, prove that $$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0.$$	**CO1**
Q10.	Solve the following differential equation to find the position vector $\vec{r}(t)$ of a moving particle at time $t > 0$. $$\frac{d^2\vec{r}}{dt^2} = -32\,\hat{k},$$ with initial conditions $\vec{r}(0) = 100\hat{k}$, and $\left.\frac{d\vec{r}}{dt}\right\|_{t=0} = 8\,\hat{\imath} + 8\,\hat{\jmath}$.	**CO5**
Q11.	Find the reduction formula for $\int \cos^n x\ dx$, where n is being positive integer and hence evaluate $\int_0^{\frac{\pi}{2}} \cos^n x\, dx$. **OR** Using reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x\ dx$, evaluate $\int_0^{\infty} \frac{dx}{(16+x^2)^{\frac{9}{2}}} dx$.	**CO2**

SECTION C (Q12a. and Q12b. both have internal choices; Each question carries 10 marks)		
Q12.	a. Find the polar equation of the ellipse with eccentricity e and semi major axis a, considering one focus of the ellipse at origin and the corresponding directrix to the right of the origin. If $a = 39$, $e = 0.25$ find the distance from the focus to the associated directrix. **OR** The coordinate axes are to rotate through an angle α to produce an equation for the curve $3x^2 + 2\sqrt{3}\,xy + y^2 - 8x + 8\sqrt{3}y = 0$ that has no cross product (xy) term. Find α and the new equation. Identify the curve. b. Providing necessary information trace the following curve. $$x^3 + y^3 = 3axy, a > 0.$$ **OR** Using y' and y'' graph the function $y = x^3(x+2)$. Include the coordinates of any local extreme points and inflection points.	**CO4**

END