Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
Online End Semester Examination, January 2020

Course: Mathematical Physics - I **Semester: I**
Program: B. Sc. (H) Physics

Time 03 hrs.
Course Code: PHYS 1011 **Max. Marks: 100**

SECTION A

1. **Each Question will carry 5 Marks**
2. **Instruction: Complete the statement / Select the correct answer(s)/Write short answers**

S. No.	**Question**	**CO**
Q1	Discuss the physical significance of curl of a vector field. What happens when the curl of a vector vanishes? (Describe in words).	**CO3**
Q2	The local maxima and minima of $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$ are: a) $x = 0$ is local maxima and $x = 1, -2$ are local minima b) $x = -1$ is local maxima and $x = 0, -2$ are local minima c) $x = 0, -1$ are local maxima and $x = -2$ is local minima d) $x = -1, -2$ are local maxima and $x = 0$ is local minima	**CO1**
Q3	The solution of exact differential equation $\left(1 + e^{\frac{x}{y}}\right) + e^{x/y}\left(1 - \frac{x}{y}\right)\frac{dy}{dx} = 0$ is: a) $2x + ye^{x/y} = c$ b) $x + ye^{x/y} = c$ c) $2x - ye^{x/y} = c$ d) $x - ye^{x/y} = c$	**CO3**
Q4	What would be the solution of the first order linear differential equation $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$? a) $\frac{y}{x+1} = e^x + c$ b) $y = xe^x + c$ c) $\frac{y}{x-1} = e^{-x} + c$ d) $= xe^{-x} + c$	**CO2**
Q5	The area of a triangle having vertices at P(1, 3, 2), Q (2, -1, 1), R(-1, 2, 3) is: a) $\sqrt{107}$ b) $\frac{1}{2}\sqrt{107}$ c) $\sqrt{117}$ d) $\sqrt{105}$	**CO1**
Q6	The Wronskian of $\frac{d^2y}{dx^2} + 4y = \tan 2x$ is: a) $\sin 2x$ b) 2 c) $4\cos 2x$ d) 4	**CO2**

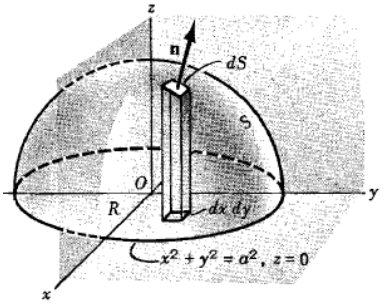
SECTION B

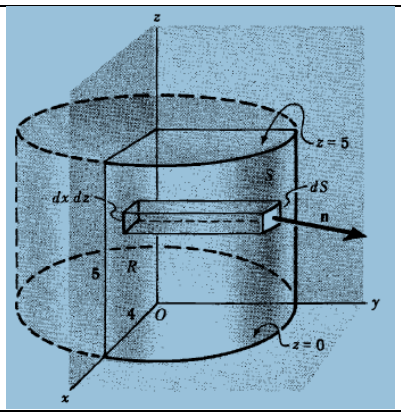
1. **Each question will carry 10 marks**
2. **Instruction: Write short / brief notes**

Q7	Find the solution of the following 2nd order linear differential equation: $(D^2 - 4D + 4)y = 8x^2 e^x \cos 2x$	**CO2**
Q8	Define orthogonal curvilinear coordinate system. If (u_1, u_1, u_3) is a set of curvilinear coordinates, write an expression for the arc length in this coordinate system. Derive the arc length expressions in cylindrical and spherical polar coordinates.	**CO3**
Q9	a) Define Dirac delta function. List three important properties of Dirac delta function (**4 Marks**) b) Using Lagrange Multiplier's method, compute the maxima/minima of the function (**6 Marks**) $f(x, y, z) = x^2 - y^2$ on the surface $x^2 + 2y^2 + 3z^2 = 1$	**CO1**
Q10	Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, and $\vec{a}$ is a constant vector ($\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$). Prove that $\vec{\nabla} \cdot \left(\frac{\vec{a} \times \vec{r}}{r^n}\right) = 0$	**CO3**
Q11	Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. If an object is moving in this field from (1,-2,1) to (3,1,4), find the work done. **OR** Find the directional derivative of A^2, where $\vec{A} = xy^2\hat{\imath} + zy^2\hat{\jmath} + xz^2\hat{k}$, at the point (2,0,3) in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 14$ at the point (3,2,1).	**CO4**

Section C

1. **Each Question carries 20 Marks.**
2. **Instruction: Write long answer.**

Q12	a) State Gauss divergence theorem and discuss its physical significance. (**5 marks**) b) If $\vec{F} = y\hat{\imath} + (x - 2xz)\hat{\jmath} - xy\hat{k}$, evaluate $\iint (\vec{\nabla} \times \vec{F}).\hat{n}\, dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy plane (see the figure below). (**15 marks**)  **OR** a) State Stokes' theorem and discuss its physical significance. (**5 marks**) b) Evaluate $\iint \vec{A} \cdot \hat{n}\, dS$, where $\vec{A} = z\hat{\imath} + x\hat{\jmath} - 3y^2z\hat{k}$ and S is the surface if the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$ (see the figure below). (**15 marks**)	**CO4**

		 z z = 5 S dS dx dz n 5 R 4 O y z = 0 x		