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| **Name:**<br>**Enrolment No:** | UPES<br>UNIVERSITY WITH A PURPOSE |

**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**
**End Semester Examinations (Online Mode), June 2021**

**Course:** Differential Equations **Semester:** II
**Program:** B.Sc Mathematics **Time:** 3 Hrs
**Course Code:** MATH1031 **Max. Marks:** 100

**SECTION - A** **6 x 5 = 30 Marks**

**1. Each Question will carry 5 Marks**
**2. Instruction: Type the correct option(s)**

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| Q 1 | The function $y = ax + be^x$ where $a$ and $b$ are arbitrary constants is the solution of the differential equation<br>A. $(x+1)y'' + xy' + y = 0$ B. $(x-1)y'' - xy' + y = 0$<br>C. $(x-1)y'' + xy' - y = 0$ D. None of these | CO1 |
| Q 2 | For the differential equation $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$, the integrating factor is given by<br>A. $e^x$ B. $\frac{1}{x^2}$<br>C. $\frac{1}{y^2}$ D. None of these | CO1 |
| Q 3 | The solution of the equation $\frac{dy}{dx} = e^{2x-y} + x^3e^{-y}$ is<br>A. $e^y = \frac{e^{2x}}{2} + \frac{x^4}{4} + c$ B. $e^y = \frac{e^{2x}}{2} - \frac{x^4}{4} + c$<br>C. $e^x = \frac{e^{2y}}{2} + \frac{x^4}{4} + c$ D. None of these | CO2 |
| Q 4 | The velocity of a chemical reaction is given by $\frac{dx}{dt} = k(a-x)$ where $x$ is the amount transferred in time $t$, $k$ is a constant and $a$ is the concentration at time $t = 0$ when $x = 0$. Then the value of $x(t)$ is<br>A. $a(1-e^{-kt})$ B. $k(1-e^{-kt})$<br>C. $a(1+e^{-kt})$ D. None of these | CO4 |
| Q 5 | The solution of the exponential growth model $\frac{dN}{dt} = rN, N(0) = n_0$ where $r > 0$ is given by<br>A. $n_0e^{rt}$ B. $n_0e^{-rt}$<br>C. $n_0te^{rt}$ D. None of these | CO4 |

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| Q 6 | For the linear autonomous system $\frac{dx}{dt} = -x, \frac{dy}{dt} = 2x - 2y$, the equilibrium point $\boldsymbol{X}$=0 is<br>A. Asymptotically stable B. Unstable<br>C. Center D. None of these | CO5 |
| **SECTION – B** **10 x 5 = 50 Marks**<br>**1. Each question will carry 10 marks**<br>**2. Instruction: Answer on a separate white sheet, scan and upload the solutions.** | | |
| Q 7 | Write a short notes on Mathematical modeling and explain characteristics of mathematical models. | CO4 |
| Q 8 | Solve the Cauchy-Euler equation $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + lnx$ | CO3 |
| Q 9 | Define exact differential equation and prove that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. | CO2 |
| Q10 | Check whether the equation $y(x^2y^2 + 2)dx + x(2 - 2x^2y^2)dy = 0$ is exact or not and solve the equation by suitable technique. | CO2 |
| Q 11 | Find all equilibrium solutions of the system of nonlinear differential equations $\frac{dx}{dt} = 1 - xy, \frac{dy}{dt} = x - y^3$ and determine whether they are stable or unstable. | CO5 |
| **Section – C** **1 x 20 = 20 Marks**<br>**1. Each Question carries 20 Marks.**<br>**2. Instruction: Answer on a separate white sheet, scan and upload the solutions.** | | |
| Q 12 | Derive the method to find the general solution of $y'' + Py' + Qy = R$ by changing the dependent variable and removing the first derivative. Using this method solve the equation $y'' - \frac{2}{x}y' + \left(1 + \frac{2}{x^2}\right)y = xe^x, x > 0$ **[20 Marks]**<br>**(OR)**<br>(a): Apply the method of variation of parameters to solve the differential equation $(D^2 + 1)y = cosec\, x.\cot x$ **[10 Marks]**<br>(b) Solve $x^2y'' - 2x(1 + x)y' + 2(1 + x)y = x^3$ by obtaining a part of the complimentary function. **[10 Marks]** | CO3 |