Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
Online End Semester Examination, May 2021

Course: Metric Spaces & Complex Analysis **Semester: VI**
Course Code: MATH 3005 **Time: 03 hrs.**
Programme: B.Sc. (Hons.) Mathematics **Max. Marks: 100**

SECTION - A **6 X 5 = 30 Marks**

1. Each Question will carry 5 Marks
2. Instruction: Select the correct option(s)

Q 1	Let (X, d) be a metric space. (X, d) is disconnected if there exists a non-empty proper subset of X A. Both open and closed. B. Open C. Closed D. Neither open nor closed.	CO2
Q 2	Every convergent sequence is a A. Cauchy Sequence B. Bounded Sequence C. Unbounded Sequence D. None of these	CO1
Q 3	If $f(z) = \frac{z^2}{(z+2)(z-1)^2}$, then residue of $z = -2$ is: A. 5/9 B. 4/9 C. 1/9 D. 3/9	CO3
Q 4	The value of $\int_{-1+i}^{1+i} z^2 dz$ along the parabola: $x = t, y = t^2, -1 \leq t \leq 1$ is: A. -4/3 B. 3/4 C. 1 D. -1	CO4
Q 5	The value of m so that $2x - x^2 + my^2$ may be harmonic is: A. 0 B. 1 C. 2 D. 3	CO3
Q 6	The radius of convergence of the power series $\sum \frac{2+in}{2^n} z^n$ is: A. 1 B. 2 C. 0 D. ∞	CO3

SECTION – B **10 X 5 = 50 Marks**

1. Each question will carry 10 marks
2. Instruction: Answer on a separate white sheet, upload the solution as image.

Q 1	Let (X, d_1) and (Y, d_2) be two metric spaces. A function $f : X \rightarrow Y$ is continuous on X if and only if for each open set $G \subset Y$, $f^{-1}(G)$ is an open subset of X.	CO1
Q 2	Prove that every contraction mapping T on a complete metric space (X, d) has a unique fixed point.	CO5
Q 3	If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z by Milne Thomson method.	CO3

Q 4	Show that $w = \frac{i-z}{i+z}$ maps the real axis of z plane into the circle $\|w\| = 1$ and the half plane $y > 0$ into the interior of unit circle $\|w\| = 1$ in the w plane.	CO3
Q 5	Evaluate $\oint_c \frac{1}{Z^2\, sinZ}\, dz$ where C is the triangle with vertices (0, 1), (2, -2), (7, 1).	CO4

Section – C **1 X 20 = 20 Marks**

1. Each Question carries 20 Marks.

2. Instruction: Answer on a separate white sheet, upload the solution as image.

Q 1	Using complex variable techniques, evaluate the integral $\int_0^{2\pi} \frac{sin^2\theta - 2cos\theta}{2+cos\theta}\, d\theta$. **OR** Using complex variables, Evaluate the real integral $\int_0^{\infty} \frac{cos\, 3x}{(x^2+1)(x^2+4)}\, dx$.	CO4