UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2021

Course: Mathematical Physics II | **Semester: III**
Program: B.Sc. Physics (H) | **Duration: 03 hrs.**
Course Code: PHYS 2001 | **Max. Marks: 100**

Instructions:

- There are three Sections (Section A, Section B and Section C).
- Section A: All the questions are compulsory.
- Section B: one question has internal choice.
- Section C: one question has internal choice.

SECTION A $(\mathbf{5Q \times 4M = 20\ Marks})$

S. No.		Marks	COs
Q1.	Define Isomorphism and Homomorphism with examples.	**4**	**CO1**
Q2.	Determine regular singular points of the differential equation $2x^2y'' + 3xy' + (x^2 - 4)\,y = 0.$	**4**	**CO2**
Q3.	Authenticate the recurrence relations of Bessel function $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x).$	**4**	**CO2**
Q4.	Validate $\Gamma(n+1) = n\Gamma(n)$, where Γ is a gamma function.	**4**	**CO3**
Q5.	Assess the Auxiliary equations of one-dimensional wave equation.	**4**	**CO4**

SECTION B $(\mathbf{4Q \times 10M = 40\ Marks})$

Q6.	Estimate the values of [22, 1] and [13, 3], using Christoffel symbols if $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2 \sin^2\theta\,(d\varphi)^2$	**10**	**CO1**
Q7.	Validate for the function $f(x)$, for which the nth derivative is continuous and $P_n(x)$ is the Legendre polynomial of degree n. $\int_{-1}^{1} f(x)\,P_n(x)dx = \frac{(-1)^n}{2^n\,n!}\int_{-1}^{1}(x^2-1)^n\,f^n(x)\,dx$	**10**	**CO2**
Q8.	Establish the relation between beta and gamma function as	**10**	**CO3**

	$\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ **OR** Evaluate $\int_{-1}^{1} (1+x)^{p-1}(1-x)^{q-1}\,dx$ using gamma function.		
Q9.	Determine the extended power series solution of the differential equation $x^2y'' + 4xy' + (x^2+2)\,y = 0.$	**10**	**CO2**
	SECTION-C	**($2Q \times 20M = 40$ Marks)**	
Q10.	(a) A covariant tensor has components $xy, 2y - z^2, xz$ in rectangular coordinates. Find its covariant components in spherical coordinates. (b) Prove that Bessel function, $J_n(x)$ is the coefficient of z^n in the expansion of $e^{\frac{x}{2}\left(z-\frac{1}{z}\right)}$.	**10** **10**	**CO1** **CO2**
Q11.	Articulate one-dimensional equation for a stretched string and solve it via the method of separation of variables. **OR** (a) Formulate two-dimensional equation for a rectangular membrane. (b) Solve the Laplace's equation in polar coordinates $r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$ using the method of separation of variables.	**20** **10** **10**	**CO4** **CO4** **CO4**