Name: **Enrolment No:** **SAP ID:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2021

Course: Group Theory I **Semester: III**
Program: BSc. (Hons) Mathematics
Time: 3 hrs. **Course Code: MATH 2028**
Max. Marks: 100
All questions are compulsory.

SECTION A

Instructions:
Each question will carry 4marks

Q 1	Show that every cyclic group is an abelian group.	**4M**	**CO2**
Q 2	Let G be a group and let $a \in G$ be of finite order n. Then for any integer k, prove that order of $a^k = \frac{x}{(n,k)}$ where (n,k) denotes the H.C.F of n and$k$.	**4M**	**CO2**
Q 3	Determine whether the groups $G = (\{0,1,2,3\}, +_4)$ and $G' = (\{1,2,3,4\}, \times_5)$ are isomorphic or not.	**4M**	**CO5**
Q 4	Let S be non-empty set and $P\ (S)$ be the collection of all subsets of S. Let the binary operation Δ called the symmetric difference of sets be defined as $A \,\Delta\, B = (A - B) \cup (B - A)\ \forall\, A, B \in P(S)$ then prove that $(P\ (S), \Delta)$ is an abelian group .	**4M**	**CO1**
Q5	If H is a subgroup of G and N is a normal subgroup of G, then show that $H \cap N$ is a normal subgroup of H.	**4M**	**CO3**
	SECTION B **Instructions:** **Each question will carry 10 marks**		
Q1	Suppose G is a group and N is a normal subgroup of G. Let $f: G \longrightarrow G/N$ defined by $f(x) = Nx \quad \forall\, x \in G$. Then prove that f is a homomorphism of G onto G/N and kernel of $f = N$.	**10M**	**CO3**
Q2	Prove that the necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup is that $a \in H, b \in H \Rightarrow ab^{-1} \in H$ where b^{-1} is the inverse of b in G.	**10M**	**CO2**
Q3	Show that the order of each subgroup of a finite group is a divisor of the order of the group.	**10M**	**CO4**
Q4	Prove that the set $G = \{1,2,3,4,5,6\}$ is a finite abelian group of order 6 with respect to multiplication modulo 7. **OR** Define Dihedral group. Find the group of symmetries of a square.	**10M**	**CO1**

	SECTION C **Instructions:** **Each question will carry 20 marks**		
Q1	i. Show that the multiplicative group $G = \{1, -1, i, -i\}$ is isomorphic to the permutation group $G' = \{I, (abcd), (ac)(bd), (adcb)\}$ on four symbols a, b, c, d. ii. Let R_+ be the multiplicative group of all positive real numbers and R be the additive group of all real numbers. Show that the mapping $g: R_+ \longrightarrow R$ defined by $g(x) = \log x \ \forall x \in R_+$ is an isomorphism. **OR** Prove the following results i. If H be a normal subgroup of a group G and K is normal subgroup of G containing H, then $G/K \cong (G/H)/(K/H)$. ii. Let G be a group and let H be any subgroup of G. If N is any normal subgroup of G, then $(HN)/N \cong H/(H \cap N)$.	**(10+10)M**	**CO5**
Q2a. **Q2b.**	If H, K are two subgroup of a group G, then prove that HK is a subgroup of G *iff* $HK = KH$ Suppose that N and M are two normal subgroup of G and $N \cap M = \{e\}$. Show that every element of N commutes with every element of M.	**10M** **10M**	**CO3** **CO3**