

Name: Enrolment No:	
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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Matrices **Semester: I**
Program: B.Sc. (Hons.) (Physics/Geology/Chemistry) **Time: 03 hrs.**
Course Code: MATH 1029 G **Max. Marks : 100**

Instructions: Attempt all the questions. Q9 and Q11 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Express the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices.	4	CO1
Q2	Define the Inverse of a square matrix and hence find the inverse of $A = \begin{bmatrix} 1 & 5 & -2 \\ 3 & -1 & 4 \\ -3 & 6 & -7 \end{bmatrix}$.	4	CO2
Q3	Define Linear dependency and independency of vectors. Find the condition on "a" for which the set $S = \{(0,1, a), (a, 1,0), (1, a, 1)\}$ is linearly independent.	4	CO3
Q4	For the transformation $\xi = x \cos \alpha - y \sin \alpha$; $\eta = x \sin \alpha + y \cos \alpha$, prove that the coefficient matrix A is orthogonal. Hence write the inverse transformation.	4	CO4
Q5	Find the characteristic polynomial of $A = \begin{bmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}$.	4	CO5

SECTION B
(4Qx10M= 40 Marks)

Q6	If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, show that $A(adj A) = (adj A)A = A I$.	10	CO1
Q7	Solve the system $x + y + z = 5$; $x + 2y + 2z = 6$; $x + 2y + 3z = 8$ using Crout's decomposition technique.	10	CO3
Q8	Solve the system $x + 2y + 3z = 5$; $2x + 8y + 22z = 6$; and $3x + 22y + 82z$ using an appropriate LU decomposition technique.	10	CO3

Q9	<p>State the Cayley Hamilton Theorem. Verify the Cayley Hamilton Theorem for $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and hence find A^{-1}.</p> <p style="text-align: center;">OR</p> <p>Define the minimal polynomial of a matrix. If $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$, find its minimal polynomial.</p>	10	CO4
SECTION-C (2Qx20M=40 Marks)			
Q10	<p>(a) Solve the system $\begin{bmatrix} 2 & -7 & 4 \\ 1 & 9 & -6 \\ -3 & 8 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 1 \\ 6 \end{bmatrix}$ using Gauss-Jordan technique.</p> <p>(b) Find the non-trivial solutions of the following system of equations using the concept of rank.</p> $\begin{aligned} 2x + y + 2z &= 0 \\ x + y + 3z &= 0 \\ 4x + 3y + 8z &= 0 \end{aligned}$	20	CO2
Q11	<p>Diagonalize the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.</p> <p style="text-align: center;">OR</p> <p>Prove that the eigen vectors of $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ are not orthogonal.</p>	20	CO4