Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Engineering Mathematics-I **Semester: I**
Program: B. Tech SOE **Time: 03 hrs.**
Course Code: MATH 1049 **Max. Marks: 100**

Instructions: Attempt all questions

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Examine the consistency of the system and if it is consistent, solve the equations: $2x - y + z = 9; 3x - y + z = 6; 4x - y + 2z = 7;\ -x + y - z = 4.$	4	CO1
Q 2	Evaluate $\int_0^3 \int_0^1 (x^2 + 3y^2)dxdy$	4	CO2
Q 3	If $f(cx - az, cy - bz) = 0$ then show that $a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = 0.$	4	CO2
Q 4	Find the divergence and curl of the vector $\vec{F}(x, y, z) = xz^3\,\hat{\imath} - 2x^2yz\,\hat{\jmath} + 2yz^4\hat{k}$.	4	CO3
Q 5	A vector field is given by $\vec{F} = (\sin y)\hat{\imath} + x(1 + \cos y)\hat{\jmath}$. Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2, z = 0$.	4	CO3

SECTION B
(4Qx10M= 40 Marks)

Q 6	Show that the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is similar to its diagonal matrix. Also, find its modal matrix.	10	CO1

Q 7	Change the order of the integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\ dy\, dx$ and hence evaluate the same.	**10**	**CO2**
Q 8	If $u = x + y + z, v = x^2 + y^2 + z^2, w = yz + zx + xy$, prove that $grad\ u, grad\ v$ and $grad\ w$ are coplanar vectors.	**10**	**CO3**
Q 9	Expand $f(x, y) = e^x \cos y$ in the powers of x and y by using Maclaurin's series. **OR** Expand $f(x) = x$ in a half-range sine series in the interval $(0, 2)$.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Find the Fourier series of $f(x) = x + x^2$ in the interval $(-\pi, \pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ … …	**20**	**CO4**
Q11 A	Evaluate $\iint \vec{F} \cdot d\vec{S}$ using Gauss's divergence theorem, where $\vec{F} = 2xy\hat{\imath} + yz^2\hat{\jmath} + zx\hat{k}$ and S is the surface of the region bounded by $x = 0, y = 0, z = 0, y = 3, x + 2z = 6$. **OR** If $\vec{A} = 2xz\hat{\imath} - x\hat{\jmath} + y^2\hat{k}$, evaluate $\iiint_V \vec{A}\ dv$, where V is the region bounded by the surface $x = 0, y = 0, x = 2, y = 6, z = x^2, z = 4$.	**10**	**CO3**
Q11 B	Prove that $(y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational. **OR** If $u = x^2 + y^2 + z^2$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then find $div(u\vec{r})$ in terms of u.	**10**	