Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Computational Gas Dynamics **Semester: I**
Program: M. Tech. CFD **Time : 03 hrs.**
Course Code: ASEG 7020 **Max. Marks: 100**

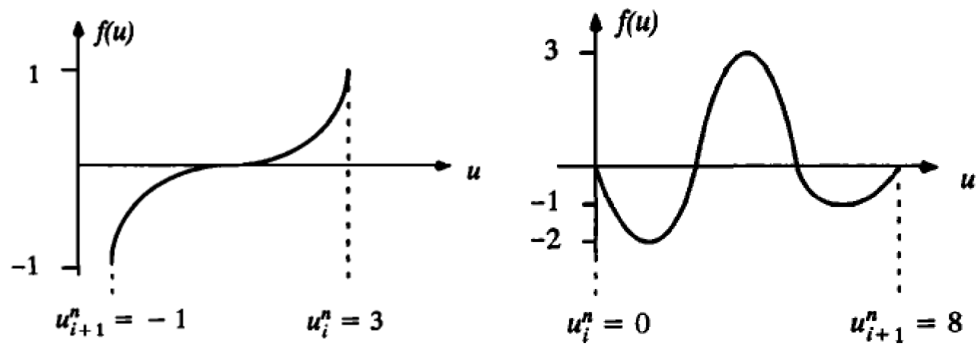
Instructions: Assume missing data, if any, appropriately.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Discuss the conditions for occurrence of the following in the solution of unsteady, one-dimensional Euler equations a) Compression wave b) Expansion wave	**04**	**CO1**
Q 2	Illustrate a first order upwind method for the linear advection equation using wave speed splitting.	**04**	**CO3**
Q 3	Define a Riemann problem for inviscid Burger's equation. Illustrate a typical solution on an *x*-*t* plane.	**04**	**CO2**
Q 4	Draw wave diagram of right running compression wave and discuss the Rankine-Hugoniot condition for shock speed.	**04**	**CO2**
Q 5	List down the various non-linear stability conditions for the solution of one-dimensional Euler equations.	**04**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

Q 6	The 1-D unsteady Euler equations, in primitive variable form, are given by $$\boldsymbol{U}_t + [A]\boldsymbol{U}_x = 0$$ where $$\boldsymbol{U} = [\rho, u, p]^T$$	**10**	**CO1**

	and $$[A] = \begin{bmatrix} u & \rho & 0 \\ 0 & u & 1/\rho \\ 0 & \rho a^2 & u \end{bmatrix}$$ Find the eigenvalues for this system of equations.		
Q 7	Consider the scalar flux function illustrated below. Find the conservative numerical flux $f_{i+1/2}^{n}$ of Roe's first order upwind method. 	**10**	**CO4**
Q 8	Write down the expressions for the conservative numerical fluxes for the following numerical schemes for the solution of 1D scalar wave equation: a) FTBS b) FTCS	**10**	**CO3**
Q 9	Derive the characteristic/wave form of the 1-dimensional, unsteady Euler equations that governs the propagation along the characteristic $\frac{dx}{dt} = u + a$.	**10**	**CO1**
	SECTION-C **(2Qx20M=40 Marks)**		
Q 10	Consider a Riemann problem for the unsteady one-dimensional Euler equations with p_L= 10, 000 N/m^2, ρ_L=1 kg/m^3, u_L=120 m/s and p_R=1,000 N/m^2, ρ_R = 0.125 kg/m^3, u_R = 20 m/s. Find the pressure ratio across the shock wave using Roe's approximate Riemann solver. ***OR***	**20**	**CO2**

	Consider a Riemann problem for the unsteady one-dimensional Euler equations with p_L= 10, 000 N/m^2, ρ_L=1 kg/m^3, u_L=120 m/s and p_R=1,000 N/m^2, ρ_R = 0.125 kg/m^3, u_R = 20 m/s. Find the pressure ratio across the shock wave using an exact Riemann solver.		
Q 11	The split-coefficient matrix method "splits" the system of equations $$U_t + [A]E_x = 0$$ into the following non-conservation form: $$U_t + [A^+]U_x + [A^-]U_x = 0$$ where $$[A^+] = [T][\lambda^+][T]^{-1}$$ $$[A^-] = [T][\lambda^-][T]^{-1}$$ If this method is applied to the system of equations $$U = \begin{bmatrix} u \\ v \end{bmatrix} \qquad A = \begin{bmatrix} 0 & c \\ c & 0 \end{bmatrix}$$ where c is a constant, find the following quantities: a) $[\lambda^+]$, $[\lambda^-]$ b) $[T]$, $[T]^{-1}$ c) $[A^+]$, $[A^-]$	**20**	**CO4**