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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, Dec 2022

Course: Complex Analysis	Semester: III
Program: B. Sc. (Hons.) Mathematics + Int. BSc-MSc Mathematics	Time: 03 hrs.
Course Code: MATH 2049	Max. Marks: 100

Instructions: All questions are compulsory. There is an internal choice in Q9 and Q11 only.

SECTION A
(5Qx4M=20Marks)

S. No.	Question	Marks	CO
Q1	Find the largest subset of the set of complex numbers \mathbb{C} on which the function $f(x + iy) = x^2 + iy$ is analytic.	4	CO1
Q2	Does there exist a harmonic conjugate of the function $u(x, y) = x^2 + y^2$? Justify your answer.	4	CO1
Q3	For what choices of $z_1, z_2 \in \mathbb{C}$ the equality $Ln(z_1 z_2) = Ln(z_1) + Ln(z_2)$ fails to hold?	4	CO2
Q4	Prove that $\left \oint_C \frac{\sin z}{z} dz \right \leq 2\pi e$ where C is the unit circle $ z = 1$.	4	CO2
Q5	Prove that there does not exist an entire onto function $f(z) = u + iv$ such that $u > 0$ for all $z \in \mathbb{C}$.	4	CO3

SECTION B
(4Qx10M= 40 Marks)

Q 6	Discuss the existence of $\lim_{z \rightarrow 0} e^{\left(1 + \frac{1}{z^4}\right)}$ using suitable paths passing through the point $z = 0$.	10	CO1
Q7	Evaluate $\oint_C 1 + z ^2 dz$ where C is the unit circle $ z = 1$ oriented counterclockwise.	10	CO2
Q8	Use Laurent series expansion by defining a suitable annular open connected set $r < z - 1 < R$ to comment correctly on the nature of singularity for $f(z) = \frac{z}{z^2 - 1}$ at the point $z = 1$.	10	CO3
Q9	Determine the value of $k \in \mathbb{Z}_{>0}$ so that $\frac{1}{2\pi i} \oint_C \frac{z^2 - z - k}{z - k} dz = 0$ where C is an arbitrary closed simple curve enclosing the point $z = k$ on complex plane.	10	CO4

	OR		
	Evaluate $\oint_C \frac{\sin z \cos^2 z + z^{2022}}{e^z} dz$ where C is an arbitrary closed simple curve on complex plane.		
SECTION-C (2Qx20M=40 Marks)			
Q10	<p>Consider $f(z) = \frac{e^{\frac{1}{z}}}{1 - \cos z}$.</p> <p>(i) Determine all the singularities of $f(z)$.</p> <p>(ii) Discuss the nature of $f(z)$ at $z = 0$.</p> <p>(iii) Find the value of $\oint_C z^2 f(z) dz$ where C is $z = 1$ counterclockwise.</p> <p>(iv) Find the order of poles at $z = 2\pi k, k \in \mathbb{Z} \setminus \{0\}$.</p>	20	CO3
Q11	<p>Evaluate the real integral</p> $\int_0^{\infty} \frac{\sin mx}{x} dx, (m > 0)$ <p>by clearly showing how the value of the integral $\int \frac{e^{imz}}{z} dz \rightarrow 0$ along the semicircular arc in upper half complex plane.</p> <p style="text-align: center;">OR</p> <p>Find the principal value of the real integral</p> $\int_{-\infty}^{\infty} \frac{\sin x}{x(x^2 - x + 2)} dx,$ <p>by clearly showing how the value of the integral $\int \frac{e^{iz}}{z(z^2 - z + 2)} dz \rightarrow 0$ along the semicircular arc in upper half complex plane.</p>	20	CO4

