Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, Dec. 2022

Course: **Mathematical Physics -II**
Semester: **III**
Program: BSC-H-PHYSICS-III.Int-BSC-MSC-PHYSICS-III.VR_B_2845 **Time : 03 hrs.**
Course Code: PHYS 2024 **Max. Marks: 100**

Instructions:

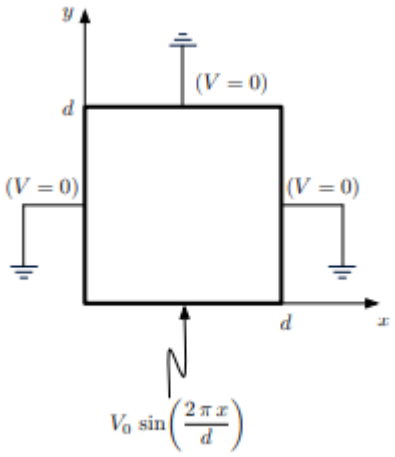
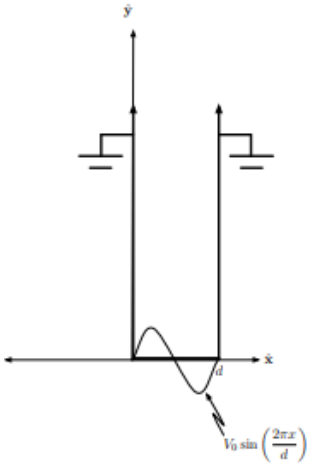
SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
	Attempt all Questions (Short answer type)		
Q.1	Prove that all the roots of the equation $Z^4 = 1$ form an Abelian Group with 'Algebraic multiplication' as the operation; where Z is a complex number,	**04**	**CO1**
Q.2	Let G be a Group and H_1 and H_2 are subgroups of G; such that $H1 \subset H_2$ and $H_2 \subset H_1$(no element of H_1 is in H_2 and vice versa); prove that $H_1 U H_2$ is never a subgroup of G.	**04**	**CO4**
Q.3	How many independent components are there in a symmetric tensor of rank 4. The dimension of the space is 4.	**04**	**CO4**
Q.4	Discuss singularity in an Ordinary Differential Equation of the form $P_1(x).y'' + P_2(x).y' + P_3(x).y = 0$; where y'' is second differential of y w.r.t. x.	**04**	**CO3**
Q.5	Find the numerical values of a) $\sqrt{(5/2)}$ b) $\beta(1/2,2)$; where $\sqrt{}$ and β are Gamma and beta functions, respectively.	**04**	**CO2**

SECTION B
(4Qx10M= 40 Marks)

	Attempt all questions. Please note that Q.9 has a choice.		

Q.6	Solve the following Ordinary Differential Equation by Frobenius method. $d^2y/dx^2 + x^2y = 0$	**10**	**CO3**
Q.7	Solve the partial differential equation by separation of variable method $\left[\frac{\partial u(x,y)}{\partial x} = 2\frac{\partial u(x,y)}{\partial y} + u\right]$; initial condition is u(x,0) = 6 exp (-3x)	**10**	**CO3**
Q.8	Evaluate the integral $$\int_{-\infty}^{\infty}(2 - 3x + 2x^2 + 5x^3)P_3(x)dx$$ Where, $P_3(x)$ is the Legendre polynomial. **HINT**: You may need to use the orthogonality condition for Legendre Polynomial	**10**	**CO3**
Q.9	**Attempt any one (Either I or II)** I: Starting from Rodrigue's formula $$H_n(x) = (-1)^n e^{x^2}\frac{d^n}{dx^n}e^{-x^2}$$ Prove the Recurrence relation: $H_{n+1}(x) = 2x.H_n(x) - 2n.H_{n-1}(x)$ **OR** II. A measurement of a physical quantity (x) gives results with probability p(x) = A. |x|; -a ≤x ≤ a; = 0; everywhere else a) Normalize p(x) and find A b) Find <x>; mean of the measurements c) Find $<x^2>$ d) Find the standard deviation of the measurements	**10**	**CO4** **CO4**
SECTION-C **(2Qx20M=40 Marks)**			
	Attempt all questions. Please note that Q. 11 has a choice.		

Q.10	 Statement: Find the Potential V(x,y) in the region surrounded by the four wires. The initial conditions are: V(x,y=0) = V_0sin(2πx/d) V(x,y=d) = 0 V(x=0,y) = 0 V(x=d,y) = 0	**20**	**CO3**
Q.11	**Attempt any one (Either I or II):** I. Use Rodrigue's Formula $$P_n(x) = \left(\frac{1}{n!.\,2^n}\right).\frac{d^n}{dx^n}\,(x^2 - 1)^n$$ to evaluate $P_n(x)$ for n = 0, 1, 2 and 3 and hence, expand the function $f(x) = x^4+2x^3+2x^2-x-3$ in terms of Legendre polynomials. **OR** **II**  Statement: Two infinite parallel plates are held at potential V=0 and are separated by a distance d. A third plate connecting these plates is kept at V(x,y=0) = V_0Sin(2πx/d); as shown in the figure below. Find V(x,y) in the space between the three plates.	**20** **20**	**CO4** **CO4**