Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Mathematics III (Numerical Methods) **Semester: III**
Program: B. Tech ASE **Time : 03 hrs.**
Course Code: MATH 2044 **Max. Marks: 100**

Instructions:

1. Section A has 5 questions. All questions are compulsory.
2. Section B has 4 questions. All questions are compulsory. Question 9 has internal choice to attempt any one.
3. Section C has 2 questions. All questions are compulsory. Question 11 has internal choice to attempt any one.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	For $r = 3h(h^6 - 2)$, find the percentage error in r at $h = 1$, if the percentage error in h is 5.	**4**	**CO1**
Q 2	If $y(25) = 0.2707,\ y(30) = 0.3027,\ y(35) = 0.3386,\ y(40) = 0.3794$, apply Gauss's forward interpolation formula to obtain $y(32)$.	**4**	**CO2**
Q 3	Find, from the following table, the area bounded by the curve $y = f(x)$ and the x −axis from $x = 7.47$ to $x = 7.52$. x: 7.47, 7.48, 7.49, 7.50, 7.51, 7.52 $f(x)$: 1.93, 1.95, 1.98, 2.01, 2.03, 2.06	**4**	**CO3**
Q 4	Given: $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}.$$ Compute the lower triangular matrix L of the Cholesky factorization method.	**4**	**CO4**

<table>
<tr><td>Q 5</td><td>Solve the boundary value problem
$$(1+x^2)y''+4xy'+2y=2,\ y(0)=0, y(1)=1/2$$
by finite difference method. Use central difference approximation with $h=1/3$.</td><td>4</td><td>CO6</td></tr>
<tr><td colspan="4">SECTION B
(4Qx10M= 40 Marks)
Instruction: Question 9 has internal choice, attempt any one.</td></tr>
<tr><td>Q 6</td><td>Find a real root correct to 4 decimal places in the interval $(0,1)$ of the equation $x=e^{-x}$ using the Newton-Raphson method.</td><td>10</td><td>CO1</td></tr>
<tr><td>Q 7</td><td>Use Lagrange's interpolation formula to fit a polynomial to the following data:
<table><tr><td>x:</td><td>-1</td><td>0</td><td>2</td><td>3</td></tr><tr><td>u_x:</td><td>-8</td><td>3</td><td>1</td><td>12</td></tr></table></td><td>10</td><td>CO2</td></tr>
<tr><td>Q 8</td><td>The table below gives the result of an observation. $y(x)$ is the observed temperature in degrees centigrade of a vessel of heating water, x is the time in minutes from the beginning of observations:
<table><tr><td>x:</td><td>1.0</td><td>1.2</td><td>1.4</td><td>1.6</td><td>1.8</td><td>2.0</td></tr><tr><td>$y(x)$:</td><td>0.0</td><td>0.1280</td><td>0.5540</td><td>1.2960</td><td>2.4320</td><td>4.000</td></tr></table>
Find the approximate rate of heating at $x=1.1$ minutes.</td><td>10</td><td>CO3</td></tr>
<tr><td>Q 9</td><td>Solve the following system of equations by Doolittle's method:
$$2x+3y+z=9$$
$$x+2y+3z=6$$
$$3x+y+2z=8.$$
OR
Use Gauss Jacobi's iterative method to solve the following system of equations:
$$8x-3y+2z=20$$
$$6x+3y+12z=35$$
$$4x+11y-z=33.$$
Perform four iterations, taking initial approximation zero.</td><td>10</td><td>CO4</td></tr>
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SECTION-C (2Qx20M=40 Marks) **Instruction:** Question 11 has internal choice, attempt any one.			
Q 10	**(a)** Given that $$\frac{dy}{dx} = x + \lvert\sqrt{y}\rvert$$ with initial condition $y = 1$ at $x = 0$. Perform four iterations of Euler's modified method to obtain the solution at $x = 0.2$, taking $h = 0.2$. **(b)** Use the Runge-Kutta fourth order method to find the value of $y(0.5)$, taking step size $h = 0.5$. Given that $$\frac{dy}{dx} = x + \lvert\sqrt{y}\rvert, y(0) = 1.$$	**10+10**	**CO5**
Q 11	Solve the boundary value problem $$u_{xx} + u_{yy} = x + y + 1, 0 \leq x \leq 1, 0 \leq y \leq 1,$$ $$u = 0 \text{ on the boundary}$$ numerically using five-point formula and Liebmann iteration for uniform mesh with mesh length $h = 1/3$. Perform only four iterations of Liebmann method for the solution. **OR** Solve by Crank-Nicolson method the one-dimensional heat equation $u_{xx} = u_t$ subject to following initial and boundary conditions $$u(x, 0) = 0, u(0, t) = 0 \text{ and } u(1, t) = t,$$ for two time steps, using step length in x −direction $h = 0.25$ and mesh ratio parameter $\lambda = 1$.	**20**	**CO6**