


Name:			
Enrolment No:			
UNIVERSITY OF PETROLEUM AND ENERGY STUDIES End Semester Examination, December 2022			
Course: Group Theory II Program: B.Sc. (Hons.) Mathematics Course Code: MATH 3022		Semester: V Time: 03 hrs. Max. Marks: 100	
Instructions: Attempt all the questions. Each question in Section A carries 4 marks. Each question in Section B carries 10 marks. Each question in Section C carries 20 marks. Internal choices are available for questions 9 and 11.			
SECTION A (5Qx4M=20Marks)			
S. No.		Marks	CO
Q 1	Consider $\beta = (123)(145)$. Write β^{99} in disjoint cycle form.	4	CO3
Q 2	Consider a group of permutation $G = \{(1), (12)(34), (1234)(56), (13)(24), (1432)(56), (56)(13), (14)(23), (24)(56)\}$. Find the stabilizer of 3 and orbit of 3.	4	CO5
Q 3	What is the order of the factor group $(Z_{10} \oplus U(10))/\langle(2, 9)\rangle$?	4	CO2
Q 4	Identify the largest order of any element in $Z_{30} \oplus Z_{20}$?	4	CO3
Q 5	Find all abelian groups (up to isomorphism) of order pq , where p and q are distinct primes.	4	CO5
SECTION B (4Qx10M= 40 Marks)			
Q 6	Prove that $U(55)$ is isomorphic to $U(75)$.	10	CO2
Q 7	Let $G = U(32)$ and $H = \{1, 31\}$. The group $\frac{G}{H}$ is isomorphic to one of $Z_8, Z_4 \oplus Z_2$, or $Z_2 \oplus Z_2 \oplus Z_2$. Determine which one by elimination?	10	CO1
Q 8	Prove that $D_3 \oplus D_4 \not\cong D_{12} \oplus Z_2$.	10	CO5
Q 9	Show that for any group G , $\frac{G}{Z(G)}$ is isomorphic to $\text{Inn}(G)$. OR State and prove the Orbit -Stabilizer theorem.	10	CO4

SECTION-C
(2Qx20M=40 Marks)

Q 10	Let G be a finite abelian group and let p be a prime that divides the order of G . Prove that G has an element of order p .	20	CO4
Q 11	(i) Find the class equation for the symmetric group S_6 . (ii) Show that there is no simple group of order 56. OR (i) Find the class equation for the symmetric group A_5 . (ii) Show that there is no simple group of order 216.	10+10	CO3