Name: Enrolment No:	UPES University of Tomorrow

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Probability Theory & Statistics **Semester:** V
Program: B.Sc. (Hons.) Mathematics **Time:** 03 hrs.
Course Code: MATH 3013D **Max. Marks:** 100

Instructions: All questions are compulsory.

SECTION A (Each question carries 4 marks)

S. No.		Marks	CO
Q1	Find the variance of the distribution. Given the first four moments of the distribution about the value 5 are 2, 20, 40 and 50.	**4**	**CO1**
Q2	Obtain the second moment about origin for the continuous random variable X whose P.D.F is given by $f(x) = \lambda e^{-x/t}$, $0 \le x < \infty, \lambda > 0$	**4**	**CO1**
Q3	A random variable X has an exponential distribution with probability density function given by $f(x) = 5e^{-5x}, for\ x > 0$ and zero elsewhere. Then find the probability that X is not less than 4.	**4**	**CO2**
Q4	If $f(x, y) = k(1-x)(1-y), 0 < x, y < 1$ represents a joint density function of random variable (X, Y) then obtain the value of k.	**4**	**CO3**
Q5	The transition probability matrix of a Markov chain $\{X_n\}, n = 1, 2, 3$ .... having three states 1, 2 and 3 is $p = \begin{matrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{matrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$ then evaluate $P\{X_1 = 2, X_0 = 2\}$	**4**	**CO5**

SECTION B (Each question carries 10 marks)

S. No.		Marks	CO
Q6	In a certain factory turning out razor blades, there is a small chance of 0.001 for any blade to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing no defective, one defective and two defective blades in a consignment of 50,000 packets.	**10**	**CO2**
Q7	Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X, Y).	**10**	**CO3**
Q8	A fair dice is thrown 720 times. Use Chebyshev's inequality to find a lower bound for the probability of getting 100 to 140 sixes.	**10**	**CO4**

<table>
<tr><td>Q9</td><td>Examine if the weak law of large numbers holds for the sequence $\{X_p\}$ of independent identically distributed random variables with $P[X_k = (-1)^{k-1}.k] = \frac{6}{\pi^2 k^2}, k = 1, 2, \ldots; p = 1, 2, \ldots..$

OR

The lifetime of a certain brand of an electric bulb may be considered a random variable with mean 1200 hours and standard deviation 250 hours. Find the probability, using central limit theorem that the average lifetime of 50 bulbs exceeds 1250 hours.</td><td>10</td><td>CO4</td></tr>
<tr><td colspan="4">SECTION-C (Each question carries 20 marks)</td></tr>
<tr><td>Q10</td><td>If $X_1, X_2, X_3, \ldots \ldots \ldots \ldots X_n$ are Poisson variate with parameter lambda is equal to 3, Use the central limit theorem to estimate $P\ (220 \leq S_n \leq 260)$, where $S_n = X_1 + X_2 + X_3 \ldots \ldots \ldots \ldots + X_n\ and\ n = 75$. Explain central limit theorem as well.</td><td>20</td><td>CO4</td></tr>
<tr><td>Q11</td><td>Calculate the coefficient of correlation and obtain the lines of regression for the following data:

<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr><tr><td>y</td><td>9</td><td>8</td><td>10</td><td>12</td><td>11</td><td>13</td><td>14</td><td>16</td><td>15</td></tr></table>
OR

The joint probability mass function of (X, Y) is given by $P\ (x, y) = k\ (2x + 3y),\ x = 0, 1, 2;\ y = 1, 2, 3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of (X + Y).</td><td>20</td><td>CO3</td></tr>
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