Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2022

Course: Discrete Mathematics **Semester: II**
Program: B. Tech CSE (All Batches) **Time : 03 hrs.**
Course Code: CSEG 1012 **Max. Marks: 100**

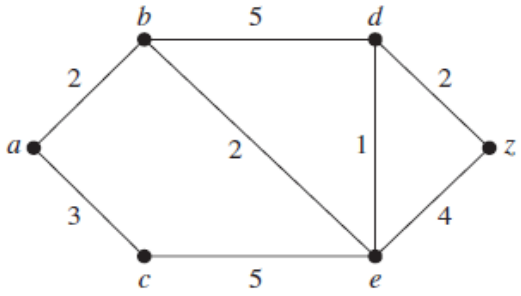
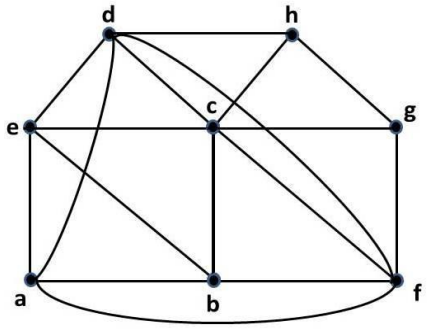
Instructions: All questions are compulsory

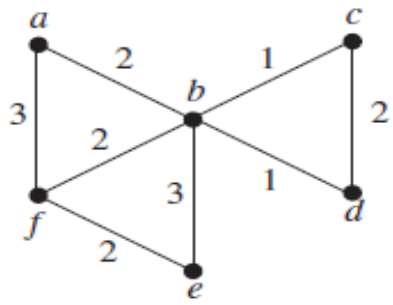
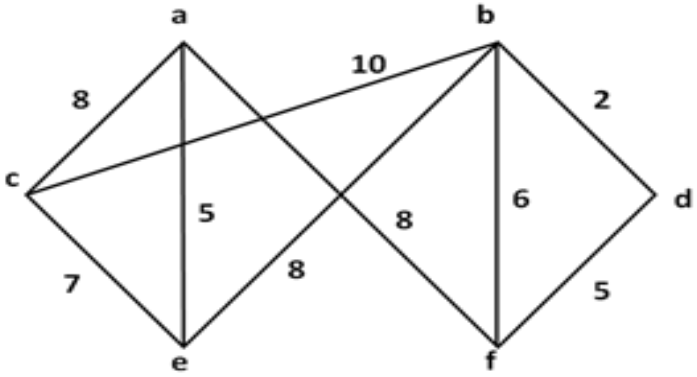
SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Over the universe of four-wheelers, let $A(x)$: x is a four wheeler, $B(x)$: x is a car and $C(x)$: x is manufactured by Tata. Express the following statements using quantifiers. i. Every four wheeler is a car ii. There are cars that are not manufactured by Tata.	**4M**	**CO2**
Q2	Is the poset $A = \{2,3,6,12,24,36,72\}$ under the relation of divisibility a Lattice? Explain.	**4M**	**CO3**
Q3	Does there exist a 4-regular graph on 6 vertices? If so construct a graph.	**4M**	**CO4**
Q4	G is a non-directed graph with 12 edges. If G has 6 vertices each of degree 3 and the rest have degree less than 3, what is the minimum number of vertices G can have?	**4M**	**CO4**
Q5	Find the generators of the cyclic group $(G, +_6)$ where $G = \{0,1,2,3,4,5\}$	**4M**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Represent the argument given below symbolically and determine whether the argument is valid If this number is divisible by 6, then it is divisible by 3.The number is not divisible by 3. Therefore this number is not divisible by 6.	**10M**	**CO2**
Q7	Let L_1 be the lattice D_6 (divisor of 6) and L_2 be the lattice $(P(S), \subseteq)$ where $S = \{a, b\}$. Then show that L_1 and L_2 are isomorphic.	**10M**	**CO3**
Q8	Using Dijkstra's algorithm, determine the length of the shortest path and hence the shortest path in the following graphs from a to z.	**10M**	**CO4**

			
Q9	Prove that the set $G = \{1,2,3,4,5,6\}$ is a finite abelian group of order 6 with respect to multiplication modulo 7. **OR** Prove that the set I of integers forms an abelian group with respect to the operation $*$ defined as $a * b = a + b + 1$	**10M**	**CO5**
SECTION-C (2Qx20M=40 Marks)			
Q 10	a. Let $f : R \rightarrow R$ be a function defined as $f(x) = 3x + 5$ and $g : R \rightarrow R$ be another function defined as $g(x) = x + 4$. Find $(gof)^{-1}$ and $f^{-1}og^{-1}$ b. Solve the difference equation $y_{n+2} - 2y_{n+1} + y_n = 3n + 4$ given that $y_0 = 0$ and $y_1 = 1$.	**20M**	**CO1**
Q11a	Define vertex colouring. Explain Welch-Powell algorithm and using this algorithm determine the coloring of the graph as shown below and hence determine the chromatic number $\chi(G)$.  **OR** Give an example of each of the following graphs **i.** Eulerian as well as Hamilton, **ii.** Eulerian but not Hamiltonian, **iii.** Hamiltonian but not Eulerian, and **iv.** Neither Eulerian nor Hamiltonian	**10M**	**CO4**

Q11b.	Using Prim's algorithm, determine a minimal spanning tree for the given weighted graph.  **OR** Using Kruskal's algorithm, determine a minimal spanning tree of the weighted graph given below.  	**10M**	**CO4**