



Name:

Enrolment No:

**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**  
**End Semester Examination, May 2022**

**Course:** Generic Elective Calculus

**Semester:** II

**Program:** B.Sc. (Hons.) Physics/Chemistry/Geology

**Time:** 03 hrs.

**Course Code:** MATH 1033

**Max. Marks: 100**

**Instructions:** Attempt all questions.

**SECTION A**  
**(5Qx4M=20Marks)**

S. No.		Marks	CO
Q 1	Use epsilon and delta definition of limit to show that the $\lim_{x \rightarrow 8} \sqrt[3]{x} = 2$	4	CO1
Q 2	Perform successive differentiation to find the nth derivate of $e^x(2x + 3)^3$	4	CO2
Q 3	For the curve $r = a \sin n\theta$ , prove that $p^2 = \frac{r^4}{n^2 a^2 - (n^2 - 1)r^2}$	4	CO3
Q 4	Expand $\left(\frac{1+e^x}{2e^x}\right)^{1/2}$ up to the term containing $x^2$ .	4	CO5
Q 5	If $u$ is a homogeneous function of two variables $x, y, z$ of degree, $n$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = nu$	4	CO6

**SECTION B**  
**(4Qx10M= 40 Marks)**

Q 1	Sketch the Cycloid $x = a(t + \sin t), y = a(1 - \cos t)$	10	CO4
Q 2	Discuss the function $x^3y^2(12 - 3x - 4y)$ for extreme values.	10	CO5
Q 3	If $x^x y^y z^z = c$ show that at $x = y = z$ $\frac{\partial^2 z}{\partial x \partial y} = -(x \log x)^{-1}$	10	CO2

Q 4	<p>State and prove Leibnitz's theorem</p> <p><b>OR</b></p> <p>Establish the formula for length of perpendicular from pole to the tangent in polar coordinates.</p>	<b>10</b>	<b>CO6</b>
<b>SECTION-C</b> <b>(2Qx20M=40 Marks)</b>			
Q 1 (a)	<p>Expand <math>\sqrt{1 + x + 2x^2}</math> in powers of <math>(x - 1)</math>.</p> <p><b>OR</b></p> <p>Verify Rolle's theorem for the function <math>f(x) = x(x + 3)e^{-x/2}</math> in <math>-3 \leq x \leq 0</math>.</p>	<b>10</b>	<b>CO5</b>
(b)	<p>Trace the Cardioid <math>r = a(1 - \cos\theta)</math></p> <p><b>OR</b></p> <p>Trace the Folium of Descartes <math>x^3 + y^3 = 3axy</math></p>	<b>10</b>	<b>CO4</b>
Q 2 (a)	<p>Discuss the asymptotes of:</p> $(x - y)^2(x^2 + y^2) - 10(x - y)x^2 + 12y^2 + 2x + y = 0$	<b>10</b>	<b>CO3</b>
(b)	<p>Evaluate <math>\lim_{x \rightarrow \infty} \sin^{-1} \sqrt{\frac{a-x}{a+x}} \operatorname{cosec} \sqrt{a^2 - x^2}</math></p>	<b>10</b>	<b>CO5</b>