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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2022

Course: Mathematics II
Program: B.Tech. (Non-CIT)
Course Code: MATH 1027

Semester: II
Time: 03 hrs.
Max. Marks: 100

Instructions:

Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (Each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 8 and 11 have internal choice.

SECTION A

S. No.		Marks	CO
Q 1	If $x^h y^k$ is an integrating factor of the non-exact differential equation $(2x^2 y^2 + y)dx - (x^3 y - 3x)dy = 0$, then find h and k .	4	CO1
Q 2	Find the solution of the differential equation $2x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 6y = 0$ ($x > 0$).	4	CO1
Q 3	Determine for what value(s) of integer n the function $u(x, y) = x^n - y^n$ is harmonic?	4	CO2
Q 4	Identify the type of singularity of the function $f(z) = e^{1/z}$ at $z = 0$.	4	CO3
Q 5	Classify the partial differential equation $x^2 u_{tt} - u_{xx} + u = 0$.	4	CO4

SECTION B

Q 6	Find the general solution of the differential equation $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 3y = 7^x - \log_e 3$.	10	CO1
Q 7	Using Cauchy's integral formula, evaluate $\int_C \frac{\sin^6 z}{(z - \frac{\pi}{6})^3} dz$, where C is the circle $ z = 1$.	10	CO2
Q 8	Apply Residue theorem to evaluate the integral $\int_C \frac{e^z - 1}{z(z-1)(z-i)^2} dz$ where C is the circle $ z = 2$. <p style="text-align: center;">OR</p> Find Taylor's and Laurent's series expansion of the function $f(z) = \frac{1}{(z+1)(z+2)^2}$ in the regions (i) $ z-1 < 2$ (ii) $ z-1 > 3$.	10	CO3

Q 9	Form a partial differential equation by eliminating the arbitrary constants a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	10	CO4
SECTION-C (2Qx20M=40 Marks)			
Q 10	<p>(i) Evaluate the real definite integral $\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)(4+x^2)} dx$ using contour integration.</p> <p>(ii) Find the bilinear transformation which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$.</p>	10+10	CO3
Q 11	<p>(i) Solve the partial differential equation $(D^2 - 2DD' + D'^2)z = \sin(2x + 3y)$, where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.</p> <p>(ii) If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3\left(\frac{\pi x}{l}\right)$, find the displacement $y(x, t)$.</p> <p style="text-align: center;">OR</p> <p>(i) Solve the first order partial differential equation $xy^2p + y^3q = (zxy^2 - 4x^3)$, by using Lagrange's method.</p> <p>(ii) Use the method of separation of variables to solve the equation $4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u$, given that $u = 3e^{-x} - e^{-5x}$ when $t = 0$.</p>	10+10	CO4