Name: **Enrolment No:**	

UPES

End Semester Examination, May 2023

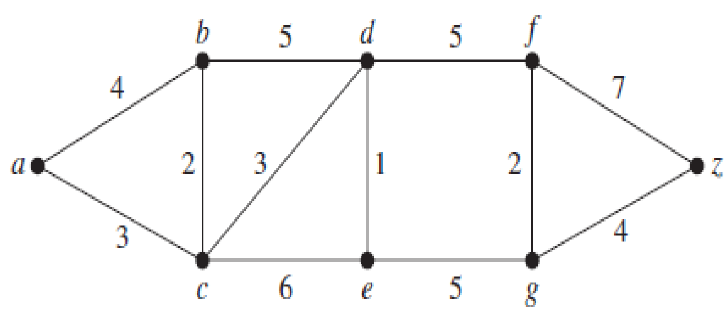
Course: **Discrete Mathematics** **Semester: II**
Program: **B. Tech. CSE** **Time: 03 hrs.**
Course Code: **CSEG 1018** **Max. Marks: 100**

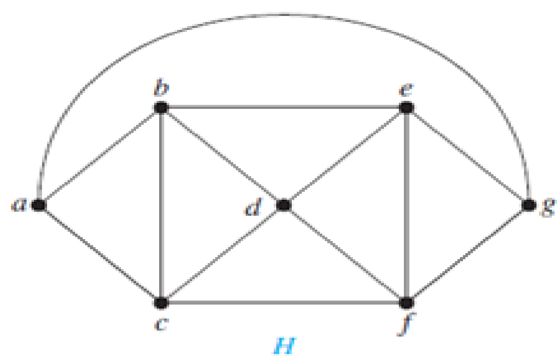
Instructions: All questions are compulsory.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q1	If p, q and r are three statements, then construct the truth table for the proposition $(p \vee q) \rightarrow (q \vee \neg r)$.	**4**	**CO2**
Q2	Show that $\neg(p \vee q)$ and $(\neg p \wedge \neg q)$ are logically equivalent.	**4**	**CO2**
Q3	Draw the Hasse diagram for the poset $(\{1,2,3,4,6,8,12\}, \mid)$, where $\mid$ represents the relation of divisibility.	**4**	**CO3**
Q4	How many generators are there in the cyclic group of order 5?	**4**	**CO5**
Q5	Prove that the cube root of unity forms an Abelian multiplicative group.	**4**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

Q6	Apply Dijkstra's algorithm to determine the length of the shortest path and hence, find the shortest path in the following graphs from $a\ to\ z$: 	**10**	**CO4**
Q7	Determine whether the given vector v is in the span of subset S of vector space V. a. $v = (2,-1,1),\ S = \{(1,0,2),(-1,1,1,)\}, S \subset \mathbb{R}^3$. b. $v = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix},\ S = \left\{\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\right\}, S \subset M_{2\times 2}(\mathbb{R})$.	**10**	**CO6**

Q8	Prove that the set of vectors $u_1 = (1,1,1)$, $u_2 = (1,2,3)$, $u_3 = (1,5,8)$ forms a basis of vector space $\mathbb{R}^3$.	**10**	**CO6**
Q9	Show that the set S of these four scalar matrices $\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}\right\}$ forms a multiplicative abelian group. **OR** Let $G = \left\{\begin{bmatrix}a & a\\a & a\end{bmatrix} : a \in \mathbb{R}, a \neq 0\right\}$. Show that G is a group under matrix multiplication.	**10**	**CO5**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	a. Let $\mathbb{N}$ be the set of all natural numbers and R be a relation on $\mathbb{N}$ defined as: xRy if and only if $x + 3y = 12$. Examine the relation for i. Reflexivity ii. Symmetricity iii. Transitivity b. If $\mathbb{N}$ denotes the set of natural numbers then solve the recurrence relation $y_{n+2} + y_{n+1} + y_n = n^2, \ \forall n \in \mathbb{N}$ *and* $n \geq 1$.	**10+10**	**CO1**
Q11	Define vertex coloring and explain Welch-Powell algorithm. Use Welch-Powell algorithm to determine the chromatic number of the following graph H:  **OR**	**20**	**CO4**

	Define chromatic polynomial and explain decomposition theorem. By using the decomposition theorem determine the chromatic polynomial and hence, find the chromatic number of the graph as shown below**:**