Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2023

Course: Linear Algebra — **Semester: II**
Program: B. Sc. (H) Mathematics — **Time: 03 hrs.**
Course Code: MATH 1047 — **Max. Marks: 100**

Instructions: Attempt all questions

S. No.		Marks	CO
	SECTION A (5Qx4M=20Marks)		
Q 1	Define the basis and dimension of a vector space.	**4**	**CO1**
Q 2	Explain linear transformation and Isomorphism.	**4**	**CO2**
Q 3	Describe Liner functional on vector space and dual space.	**4**	**CO3**
Q 4	Let λ be an eigenvalue of an invertible operator T then show that λ^{-1} is an eigenvalue of T^{-1}.	**4**	**CO3**
Q 5	In an inner product space $V(F)$, prove that $\|\alpha+\beta\|^2 = \|\alpha\|^2 + \|\beta\|^2 + 2Re\langle\alpha,\beta\rangle \; \forall \; \alpha, \beta \epsilon V$ where Re stands for the real part.	**4**	**CO4**
	SECTION B (4Qx10M= 40 Marks)		
Q 6	Prove that the linear span $L(S)$ of a non-empty subset S of a vector space $V(F)$ is the smallest subspace of the vector space $V(F)$ containing S.	**10**	**CO1**
Q 7	Find the linear map $T: \mathcal{R}^2 \to \mathcal{R}^3$ whose matrix is $A = \begin{bmatrix} 1 & -1 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}$ with basis $B = \{(1,1), (0,2)\}$ and basis $B' = \{(0,1,1), (1,0,1), (1,1,0)\}$.	**10**	**CO2**
Q 8	Let $\gamma = \beta - \frac{\langle\beta,\alpha\rangle}{\|\alpha\|^2}\alpha$ then prove the Cauchy-Schwarz inequality $|\langle\alpha,\beta\rangle| \leq \|\alpha\|\|\beta\| \; \forall \; \alpha, \beta \epsilon V$.	**10**	**CO4**

Q 9	If T is a linear transformation on $V_3(\mathcal{R})$ which is represented in the standard basis by the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Determine the eigenvalues and eigenvectors. **OR** Let W_1 and W_2 be subspaces of a finite-dimensional vector space V over a field F then prove that $(W_1 + W_2)^0 = {W_1}^0 \cap {W_2}^0$.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Find the dual basis of the basis set $B = \{(1,-1,3), (0,1,-1), (0,3,-2)\}$ for $V_3(\mathcal{R})$.	**20**	**CO3**
Q11	**(A)** Show that $\langle x, y \rangle$ is an inner product space where $\langle x, y \rangle = 2x_1\overline{y_1} + x_1\overline{y_2} + x_2\overline{y_1} + x_2\overline{y_2}, \forall x = (x_1, x_2),$ $y = (y_1, y_2) \epsilon \mathcal{R}^2(\mathcal{R})$ **(B)** Prove that every finite-dimensional vector space is an inner product space. **OR** In an inner product space $V(F)$ prove the polarization identity $\langle \alpha, \beta \rangle = \frac{1}{4}(\|\alpha + \beta\|^2 - \|\alpha - \beta\|^2 + i\|\alpha + \beta\|^2 - i\|\alpha - \beta\|^2)$ $\forall\ \alpha, \beta \epsilon V.$	**20**	**CO4**