Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2023

Course: Real Analysis II **Semester: IV**

Program: B.Sc. (H) Mathematics & Int. B.Sc. M.Sc. Mathematics **Time : 03 hrs.**

Course Code: MATH 2051 **Max. Marks: 100**

Instructions: Read all the below mentioned instructions carefully and follow them strictly:

1) Mention Roll No. at the top of the question paper.
2) Attempt all the parts of a question at one place only.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Compute by Riemann integration $\int_{-1}^{1} f(x)\,dx$, where $f(x) = \lvert x \rvert$.	**4**	**CO1**
Q 2	Determine the interval of convergence of the power series $\sum\{(1/n)\,(-1)^{n+1}(x-1)^n\}$.	**4**	**CO3**
Q 3	Give an example to show that the limit of differentials is not equal to the differential of limit.	**4**	**CO2**
Q 4	Find the interval of absolute convergence for the series $\sum_{n=1}^{\infty} x^n/n^n$.	**4**	**CO3**
Q 5	Prove that the sequence $\{f_n\}$, where $f_n(x) = nxe^{-nx^2}$, $x \geq 0$ is not uniformly convergent on $[0, k]$, $k > 0$.	**4**	**CO2**
SECTION B			
(4Qx10M= 40 Marks)			
Q 6	Prove with the help of an example that the equation $\int_a^b f'(x)dx = f(b) - f(a)$, is not always valid.	**10**	**CO1**
Q 7	Show that $\frac{1}{2}(tan^{-1}x)^2 = \frac{x^2}{2} - \frac{x^4}{4}\left(1+\frac{1}{3}\right) + \frac{x^6}{6}\left(1+\frac{1}{3}+\frac{1}{5}\right)+\ldots$, $-1 \leq x \leq 1$.	**10**	**CO3**
Q 8	Prove that the series obtained by integrating and differentiating power series term by term has the same radius of convergence as the original series.	**10**	**CO2**

Q 9	Find the radius of convergence of the series $1+\frac{a.b}{1.c}+\frac{a(a+1)b(b+1)}{1.2c(c+1)}+\cdots$. **OR** Find the radius of convergence of the series $1+x+2!\,x^2+3!\,x^3+4!\,x^4+\cdots$.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	**i)** If a function f is continuous on $[a,b]$, then there exists a number ξ in $[a,b]$ such that $\int_a^b f\,dx = f(\xi)(b-a)$. **ii)** Prove that every continuous function is integrable.	**20**	**CO1**
Q 11	**i)** State and prove Weierstrass's M test for uniform convergence. **ii)** Show that the sequence $\langle f_n(x)\rangle$, where $f_n(x)=\frac{log(1+n^3x^2)}{n^2}$ is uniformly convergent on [0, 1]. **OR** **i)** Test for uniform convergence, the series, $\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}+\cdots, \quad -\frac{1}{2}\le x\frac{1}{2}$. **ii)** If $\langle f_n\rangle$ is a sequence of continuous functions on an interval $[a,b]$ and if $f_n\to f$ uniformly on $[a,b]$, then f is continuous on $[a,b]$.	**20**	**CO2**