


| Name: | |  | |
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| Enrolment No: | | | |
| UPES End Semester Examination, May 2023 | | | |
| Course: Probability and Statistics Program: B.Sc (H) Mathematics / Int. B.Sc M.Sc Mathematics Course Code: MATH2052 | | Semester: IV Time: 03 hrs. Max. Marks: 100 | |
| Instructions: Attempt All Questions. | | | |
| SECTION A (5Qx4M=20Marks) | | | |
| S. No. | | Marks | CO |
| Q 1 | Let $\Omega = 1,2,3,4$ be a sample space. Check whether the set $\mathcal{F} = \{\phi, \Omega, \{1\}, \{1,2\}, \{3,4\}\}$ is a sigma field. If your answer is no, then find the smallest sigma field containing \mathcal{F} . | 4 | CO1 |
| Q 2 | Two dice are thrown together. What is the probability that the number obtained on one of the dice is multiple of number obtained on the other dice? | 4 | CO1 |
| Q 3 | Let X be a discrete random variable taking values in $\{-3, -2, -1, 0, 1, 2, 3\}$ such that $P[X = -3] = P[X = -2] = P[X = -1] = P[X = 1] = P[X = 2] = P[X = 3]$ and $P[X < 0] = P[X = 0] = P[X > 0]$. Find the cumulative distribution function of X . | 4 | CO1 |
| Q 4 | In a family of 5 children, what is the probability that there will be more boys than girls? (Use Binomial and assume that the probability of having a boy or a girl is $\frac{1}{2}$ each). | 4 | CO2 |
| Q 5 | If X is a random variable with $E[X] = 3$ and $E[X^2] = 13$ then determine a lower bound for $P(-2 < X < 8)$ using Chebyshev's inequality. | 4 | CO3 |
| SECTION B (4Qx10M= 40 Marks) | | | |
| Q 6 | Let X be a random variable that gives number of die tosses required to get the first 6. Then write the range of X , p.m.f. (probability mass function), mean, variance, and evaluate $P(X \geq 1.5)$, $P(0 < X < 4)$, and $P(X = 2 X < 5)$. | 10 | CO1 |
| Q 7 | <p>The following function defines a p.d.f. (probability density function) for some $\alpha > 0$?</p> $f(x) = \begin{cases} \frac{1}{\alpha x^2}, & \text{if } x > 1 \\ 0, & \text{otherwise} \end{cases}$ <p>Calculate the value of α, the corresponding CDF, and MGF. Also evaluate the expectation and variance of the corresponding random variable.</p> | 10 | CO1 |
| Q 8 | Consider two random variables X and Y with joint PMF given in the following table. | 10 | CO2 |

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|--------------------------|--|---------|---------|---------|----|-----------|------------|----|----|----|----|----|----|----|----|----|-----|----|----|----|----|----|----|----|----|----|----|-----|---|---|---|---|---|---|---|-----|---|---|----|----|----|----|----|-----------|------------|
| | | $Y = 2$ | $Y = 4$ | $Y = 5$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | $X = 1$ | $1/12$ | $1/24$ | $1/24$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | $X = 2$ | $1/6$ | $1/12$ | $1/8$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | $X = 3$ | $1/4$ | $1/8$ | $1/12$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | Calculate $P(X \leq 2, Y \leq 4)$, formulate the marginal p.m.f. of X and Y , and evaluate $P(Y = 2 X = 1)$. Are X and Y independent? | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q 9 | <p>The length of life of an instrument produced by a machine has a normal distribution with a mean of 12 months and standard deviation of 2 months. Calculate the probability that an instrument produced by this machine will last (a) less than 7 months. (b) between 7 and 12 months.</p> <p style="text-align: center;">OR</p> <p>In a normally distributed data, 31% of the items are under 45, and 8% of the items are over 64. Find the mean and variance of the distribution. Also estimate the percentage of data (a) between 55 and 65 (b) between 45 and 60.</p> | | | | | 10 | CO3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| SECTION-C | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (2Qx20M=40 Marks) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q 10 | <p>Random variables X and Y have joint continuous distribution with p.d.f.</p> $f_{XY}(x, y) = \begin{cases} c(2x + y), & \text{if } 0 < x < 1, 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$ <p>(a) Find the value of c. (b) Compute $E[X]$ and $E[Y]$. (c) Formulate $E[X Y = y]$.</p> | | | | | 20 | CO2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q 11 | <p>The data on heights of fathers and sons are given in the following table, where X is height of fathers and Y is height of son.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>X</td> <td>65</td> <td>63</td> <td>67</td> <td>64</td> <td>68</td> <td>62</td> <td>70</td> <td>66</td> <td>68</td> <td>67</td> </tr> <tr> <td>Y</td> <td>68</td> <td>66</td> <td>68</td> <td>65</td> <td>69</td> <td>66</td> <td>68</td> <td>65</td> <td>71</td> <td>67</td> </tr> </tbody> </table> <p>Evaluate the coefficient of correlation for the following data by making a table. Also construct a regression line and consequently predict the son's height for father with height 72 inches.</p> <p style="text-align: center;">OR</p> <p>Suppose that the table given below represents the data on two random variables X and Y.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>X</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> </tr> <tr> <td>Y</td> <td>9</td> <td>8</td> <td>10</td> <td>12</td> <td>11</td> <td>13</td> <td>14</td> </tr> </tbody> </table> <p>Calculate the regression coefficient and obtain the lines of regression for the following data. Then predict the Y value for $X = 4.5$. Also compute the correlation coefficient of X and Y.</p> | | | | | X | 65 | 63 | 67 | 64 | 68 | 62 | 70 | 66 | 68 | 67 | Y | 68 | 66 | 68 | 65 | 69 | 66 | 68 | 65 | 71 | 67 | X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Y | 9 | 8 | 10 | 12 | 11 | 13 | 14 | 20 | CO3 |
| X | 65 | 63 | 67 | 64 | 68 | 62 | 70 | 66 | 68 | 67 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Y | 68 | 66 | 68 | 65 | 69 | 66 | 68 | 65 | 71 | 67 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Y | 9 | 8 | 10 | 12 | 11 | 13 | 14 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |