Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2023

Course: Mathematical Physics - I **Semester: I**

Program: BSc (H) Physics with Research **Time : 03 hrs.**

Course Code: PHYS 1011 **Max. Marks: 100**

Instructions: Use of scientific calculator is allowed.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Express following points in Cartesian coordinates: a) $P(1, 60^o, 2)$ b) $T(4, \pi/2, \pi/6)$	**4**	**CO1**
Q2	Prove if the following first order differential equation is homogeneous or not: $$x \sin\frac{y}{x} dy = \left(y \sin\frac{y}{x} - x\right) dx$$	**4**	**CO2**
Q3	Given a surface $$\varphi(x, y, z) = 2x^2 + xy - z = 0$$ Find the unit normal to this surface at (1,-2, 5).	**4**	**CO3**
Q4	State Cayley-Hamilton theorem and briefly cite its importance in matrix algebra.	**4**	**CO1**
Q5	State Dirac Delta function and list its properties.	**4**	**CO1**

SECTION B

(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q6	The radial displacement in a rotating disc at a distance r from the axis is given by $$r^2 \frac{d^2u}{dr^2} + r\frac{du}{dr} - u + kr^3 = 0$$ where k is a constant. Solve the equation under the following conditions: $$u(r = 0) = 0 \;\&\; u(r = a) = 0$$	**10**	**CO2**
Q7	(**a**) Given a function: $$f(x, y, z) = e^{xy} + \log(\sin zx) - \frac{1}{yz}$$ Find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y \partial z}$ (**5 Marks**) (**b**) Solve the following differential equation: (**5 Marks**) $$(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$$	**10**	**CO3**

Q8	A scope probe in the shape of ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth atmosphere and its surface begins to heat. After one hour, the temperature at any point (x, y, z) on the surface is $T(x, y, z) = 8x^2 + 4yz - 16z + 400$. Find the hottest point on the probe surface. **OR** The pressure P at any point (x, y, z) in space is $P = 400\, xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$.	**10**	**CO3**
Q9	(a) Diagonalize the following matrix: (**5 Marks**) $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ (b) Using Cayley-Hamilton theorem, find the inverse of the following matrix: (**5 Marks**) $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$	**10**	**CO1**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	(a) Calculate the directional derivative of the function $\varphi(x, y, z) = xy^2 + yz^3$ at the point $(1, -1, 1)$ in the direction parallel to the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. (**8 Marks**) (b) Find the constants a, b, c so that the vector field $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. Find the scalar field such that $\vec{F} = \vec{\nabla}\varphi$ (**12 Marks**) **OR** (a) Find the directional derivative of $\vec{\nabla} \cdot \vec{v}$ at the point $(1,2,2)$ in the direction of the outer normal of the sphere $x^2 + y^2 + z^2 = 9$ for $\vec{v} = x^4\hat{\imath} + y^4\hat{\jmath} + z^4\hat{k}$. (**8 Marks**) (b) A fluid motion is given by $\vec{V} = (y + z)\hat{\imath} + (z + x)\hat{\jmath} + (x + y)\hat{k}$ Show that the motion is irrotational and hence find the velocity potential. (**12 Marks**)	**20**	**CO4**
Q11	(a) Find the work done in moving a particle around the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$ Under the field of the force given as $\vec{F} = (2x - y + z)\hat{\imath} + (x + y - z^2)\hat{\jmath} + (3x - 2y + 4z)\hat{k}$	**20**	**CO4**

	Is this field conservative? **(8 Marks)** (b) Evaluate $\iint \vec{A} \cdot \hat{n}\, ds$, where $\vec{A} = 18z\,\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ included in the first octant. (**12 Marks**)		