Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2023

Course: Logic and Sets — **Semester: III**
Program: B.Sc. (Hons.) Mathematics — **Time : 03 hrs.**
Course Code: MATH 2032K — **Max. Marks: 100**

Instructions: Attempt all questions

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	If p be "He is rich" and q be "He is happy". Write each statement in symbolic form using p and q. Note that "He is poor" and "He is unhappy" are equivalent to $\sim p$ and $\sim q$, respectively. (a) If he is rich, then he is unhappy. (b) He is neither rich nor happy. (c) It is necessary to be poor in order to be happy. (d) To be poor is to be unhappy.	**4**	**CO1**
Q 2	(a) Define a compound proposition with an example. (b) Write the negation of the following compound statement: *"If the determinant of a system of linear equations is zero then either the system has no solution or it has an infinite number of solutions"*.	**4**	**CO2**
Q 3	Using Venn diagram, prove that $(B - A) \cup (A \cap B) = B$.	**4**	**CO4**
Q 4	Let $U = \{a, b, c, d, e\}, A = \{a, b, d\}$ and $B = \{b, d, e\}$. Find (a) $B - A$ (b) $A - B$ (c) $B' - A'$ (d) $(A \cap B\}'$ (e) $(A \cup B\}'$.	**4**	**CO3**
Q 5	Let $f: R \to R$ and $g: R \to R$ defined by $f(x) = x^2 - 2\|x\|$, and $g(x) = x^2 + 1$. Find (a) $gof(3)$ (b) $fog(-2)$ (c) $gof(-4)$ (d) $(fog)(5)$	**4**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Let A be a set of non-zero integers and let $\approx$ be the relation on A x A defined by $(a, b) \approx (c, d)$ whenever $ad = bc$. Prove that $\approx$ is an equivalence relation.	**10**	**CO5**

Q 7	Let R_5 be the relation on the set Z of integers defined by $x \equiv y(mod\ 5)$, which reads "x is congruent to y modulo 5". Find the quotient set Z/R_5.	**10**	**CO5**
Q 8	(a) Show that contrapositive and conditional propositions are logically equivalent. (b) Prove that $(p \to q) \wedge (r \to q) \equiv (p \vee r) \to q$.	**10**	**CO2**
Q 9	Determine the validity of the following argument: $p \wedge q$ $p \to r$ $s \to \sim q$ ____________ $\sim s \wedge r$ **OR** Check the validity of the following argument: *If I like mathematics, then I will study.* *Either I don't study or I pass mathematics.* *If I don't pass mathematics, then I don't graduate.* ____________ *If I graduate, then I like mathematics.*	**10**	**CO2**
SECTION-C (2Qx20M=40 Marks)			
Q 10A	Verify whether the following compound propositions are tautologies or contradictions or contingency. (a) $(p \vee q) \wedge (\sim p) \wedge (\sim q)$. (b) $(p \to q) \leftrightarrow (\sim q \to \sim p)$.	**10**	**CO2**
Q 10B	What is principal conjunctive normal form? Using truth tables, find the principal conjunctive normal form of $(p \wedge q) \vee (\sim q \wedge r)$.	**10**	**CO2**
Q 11A	If $D = \{1,2,3,....9\}$, determine the truth value of each of the following statements. i. $(\forall x \in D),\ x+4<15,$ ii. $(\exists x \in D),\ x+4=10,$ iii. $(\forall x \in D),\ x+4 \le 10,$ iv. $(\exists x \in D),\ x+4>15.$	**10**	**CO2**

	OR Explain quantifier. Give the symbolic form of the following statements: (a) Some men are genius. (b) For every x, there exists a y such that $x^2 + y^2 \geq 100$. (c) Given any positive integer, there is a greater positive integer. (d) Everyone who likes fun will enjoy each of these plays.		
Q 11B	Discuss the five basic connectives with their truth tables. Construct the truth table for the following proposition. $[(p \vee q) \wedge \sim(\sim p) \wedge (\sim q \vee \sim r)] \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$ **OR** Using the laws of proposition algebra, check the equivalence of the propositions $p \rightarrow (q \vee r)$ and $(p \rightarrow q) \vee (p \rightarrow r)$.	**10**	**CO2**