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| **Name:**<br>**Enrolment No:** | |

**UPES**

**End Semester Examination, December 2023**

**Course:** **Ordinary Differential Equations** **Semester : III**

**Program:** **B. Sc. (H) Mathematics** **Time : 03 hrs.**

**Course Code:** **MATH2048** **Max. Marks: 100**

**Instructions:** Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (Each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 8 and 10 have internal choice.

## SECTION A

| S. No. | | Marks | CO |
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| Q 1 | Define an ordinary differential equation. Determine the order and degree of the differential equation $$\left\{\frac{d^2y}{dx^2}\right\}^{1/2} + \left\{\frac{d^2y}{dx^2}\right\}^{2/5} = 0.$$ | **4** | **CO1** |
| Q 2 | Show that $\sin 2x$ and $\cos 2x$ form a set of fundamental solutions of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$. | **4** | **CO1** |
| Q 3 | For what value of $\alpha$ and $\beta$, the following differential equation $$(\alpha xy^3 + y\cos x)dx + (x^2y^2 + \beta \sin x)dy = 0$$ is exact? | **4** | **CO2** |
| Q 4 | If $y_p(x) = x\cos 2x$ is a particular solution of the differential equation $$\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} = -4\sin 2x,$$ then find the value of $\alpha$. | **4** | **CO3** |
| Q 5 | The rate at which bacteria multiply is proportional to the instantaneous number present. If the original number doubles in 2 hours, then how long does it will triple? | **4** | **CO4** |

## SECTION B

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| Q 6 | Solve the differential equation $y = 2xp - yp^2$ and investigate whether a singular solution exists. | **10** | **CO2** |
| Q 7 | Examine whether the differential equation $$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$$ is exact or not and then solve it. | **10** | **CO2** |

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| Q 8 | Find the general solution of the differential equation<br>$$4\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 9y = 144e^{-3x}.$$<br>**OR**<br>Using the method of undetermined coefficients to solve the following differential equation:<br>$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2.$$ | **10** | **CO3** |
| Q 9 | Classify (if possible) the critical points of the following autonomous system as stable or unstable:<br>$$x' = 0.01x(100 - x - y),$$<br>$$y' = 0.05y(60 - y - 0.2x).$$ | **10** | **CO5** |
| | **SECTION-C** | | |
| Q 10 | Use the method of variation of parameters to find the general solution of the differential equation<br>$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = -x^4 \sin x.$$<br>**OR**<br>Use the variation of parameters method to show that the solution of equation $\frac{d^2y}{dx^2} + k^2y = \emptyset(x), k \neq 0$, satisfying the conditions $y(0) = 0\ and\ \left(\frac{dy}{dx}\right)_{x=0} = 0$, is $y(x) = \frac{1}{k}\int_0^x \emptyset(t) \sin k(x-t)dt$. | **20** | **CO3** |
| Q 11 | (i) A body executes damped force vibrations given by the equation<br>$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt} \sin nt.$$<br>Solve the equation for both cases when $n^2 \neq b^2 - k^2$ and $n^2 = b^2 - k^2$.<br>(ii) According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is $40^0C$ and the substance cools from $80^0C$ to $60^0C$ in 20 minutes, what will be the temperature of the substance after 40 minutes? | **10+10** | **CO4** |