


Name:			
Enrolment No:			
<b>UPES</b> <b>End Semester Examination, December 2023</b>			
<b>Course: Differential Equations</b> <b>Program: B.Sc. Hons. (Physics, Chemistry &amp; Geology)</b> <b>Course Code: MATH 1034G</b>		<b>Semester: III</b> <b>Time: 03 hrs.</b> <b>Max. Marks: 100</b>	
<b>Instructions: All questions are compulsory and there are internal choices in Q9 and in Q11.</b>			
<b>SECTION A</b> <b>(5Qx4M=20Marks)</b>			
S. No.		Marks	CO
Q1.	Find the differential equation of all circles which pass through the origin and whose centers are on the $x$ -axis.	4	CO1
Q2.	Solve the differential equation, $x(1 + p^2) = 1$ , to find its general solution. Here $p \equiv dy/dx$ .	4	CO2
Q3.	Transform $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x$ , $x > 0$ , to a differential equation with constant coefficients, and find its complementary function.	4	CO3
Q4.	Form the partial differential equation by eliminating arbitrary constants $a$ and $b$ from the given relation, $2z = (ax + y)^2 + b$ . Also classify it based on its linearity.	4	CO1
Q5.	Solve the linear partial differential equation using Lagrange's method: $2p + 3q = 1$ , where $p \equiv \partial z/\partial x$ and $q \equiv \partial z/\partial y$ .	4	CO5
<b>SECTION B</b> <b>(4Qx10M= 40 Marks)</b>			
Q6.	Use the method of variation of parameters to find the general solution of $y'' - 2y' + y = e^x \tan x$ , $x > 0$ .	10	CO3
Q7.	Find $f(z)$ such that the total differential equation, $\left\{ \frac{y^2 + z^2 - x^2}{2x} \right\} dx - ydy + f(z)dz = 0$ is integrable. Hence solve it.	10	CO4
Q8.	Use Charpit's method to find the complete solution of the given nonlinear partial differential equation, $(p + y)^2 + (q + x)^2 = 1$ .	10	CO5
Q9.	Find the characteristics of $y^2 r - x^2 t = 0$ , where $r = \frac{\partial^2 u}{\partial x^2}$ and $t = \frac{\partial^2 u}{\partial y^2}$ .	10	CO1
	<b>OR</b> Classify the given partial differential equation, $u_{xx} + u_{yy} = u_{zz}$ .		

**SECTION-C**  
**(2Qx20M=40 Marks)**

Q10.	Form the partial differential equations for the following and find their orders: a) $ax^2 + by^2 + cz^2 = 1$ , by eliminating the constants $a$ , $b$ , and $c$ . b) $z = yf(x) + xg(y)$ by eliminating the functions $f$ and $g$ .	<b>20</b>	<b>CO1</b>
Q11.	Find the characteristic curves and derive the canonical form of the second order partial differential equation, $\frac{\partial^2 z}{\partial x^2} = (1 + y)^2 \frac{\partial^2 z}{\partial y^2}.$ <p style="text-align: center;"><b>OR</b></p> Give the governing equation and appropriate boundary conditions for the one-dimensional Wave equation, assuming both the end points of the string to be fixed and the initial displacement and initial velocity are given by the functions $f(x)$ and $g(x)$ , respectively. Solve the problem using separation of variables.	<b>20</b>	<b>CO5</b>