Name: **Enrolment No:**	

UPES

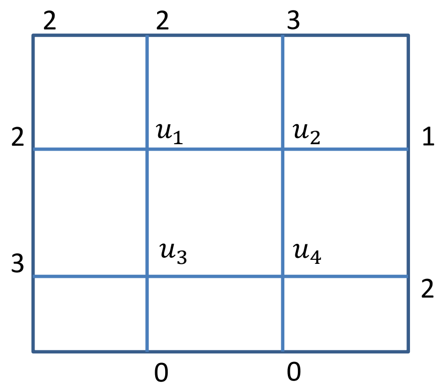
End Semester Examination, December 2023

Course: Applied Numerical Methods
Program: B.Tech AE/ME **Semester: III**
Time: 03 hrs
Course Code: MATH2053 **Max. Marks : 100**

Instructions: You must answer all of the questions. Use a scientific calculator as required for your calculations.

SECTION A (5QX4M=20 Marks)			
S. No.		**Marks**	**CO**
Q 1	Is this system of equations well-conditioned? $$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$	**4**	**CO3**
Q 2	Determine the LU decomposition for the given matrix. $$\begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}$$ Employ Cholesky's method to decompose the coefficient matrix.	**4**	**CO3**
Q 3	Present the general structure of a first-order initial value problem. Outline the standard representation of Euler's method for solving initial value problems of the first order.	**4**	**CO3**
Q 4	Write the second order difference approximations for $(i)\ y'(x_i)$ and $(ii)\ y''(x_i)$ based on central differences.	**4**	**CO4**
Q 5	Write out the diagonal five-point formula for solving (i) Laplace's equation $u_{xx} + u_{yy} = 0$ and (ii) Poisson equation $u_{xx} + u_{yy} = G(x, y)$ with uniform mesh spacing h.	**4**	**CO4**
	SECTION B (4QX10M=40 Marks)		
Q 6	Apply the LU decomposition method with the Doolittle technique for the decomposition of the coefficient matrix to solve the given system of simultaneous linear equations. $$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$	**10**	**CO3**

Q 7	Apply Newton's forward interpolation to estimate the velocity at $x = 0.4\ cm$ for a fluid near a flat surface, given the velocity distribution provided below where x represents the distance from the surface (cm) and v denotes the velocity (cm/s). Distance (x): 0.1, 0.3, 0.5, 0.7, 0.9 Velocity (v): 0.72, 1.81, 2.73, 3.47, 3.98	**10**	**CO2**
Q 8	The following system of equations is designed to determine concentrations (the c's in g/m^3) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides in g/day), $15c_1 - 3\,c_2 - c_3 = 3300$ $-3c_1 + 18c_2 - 6c_3 = 1200$ $-4c_1 - c_2 + 12c_3 = 2400$ Execute two iterations of the Gauss-Seidel method with an initial approximation set as $[c_1, c_2, c_3]^T = [0, 0, 0]$.	**10**	**CO3**
Q 9	A ball at $1200\ K$ is allowed to cool down in air at ambient temperature of $300\ K$. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by $\frac{d\theta}{dt} = -2.2067 \times 10^{-12}(\theta^4 - 81 \times 10^8), \quad \theta(0) = 1200\ K$ where θ is in K and and t in seconds. Determine the temperature at $t = 240\ s$ using the fourth order Runge-Kutta (RK) method, assuming a step size of $h = 240\ s$. ***OR*** Solve the boundary value problem $(1 + x^2)y'' + 4xy' + 2y = 2, y(0) = 0, y(1) = 1/2$ by finite difference method. Use central difference approximations with $h = 1/3$.	**10**	**CO3**
	SECTION C **(2QX20M=40 Marks)**		
Q 10	The ideal gas law is given by $pv = RT$ where p is the pressure, v is the specific volume, R is the universal gas constant, and T is the absolute temperature. This equation is only accurate for a limited range of pressure and temperature. Vander Waals came up with an equation that was accurate for larger range of pressure and temperature given by	**20**	**CO1**

	$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$ where a and b are empirical constants dependent on a particular gas. Given the value of $R = 0.08, a = 3.592, b = 0.04267, p = 10$ and $T = 300$ (assume all units are consistent), one is going to find the specific volume, v, for the above values. Without finding the solution from Vander Waals equation, what would be a good initial guess for v? Utilize Newton-Raphson method and conduct two iterations. Show all steps in calculating the estimated root, absolute relative approximate error for each iteration.		
Q 11	Solve the following Laplace equation $u_{xx} + u_{yy} = 0$ numerically, using five-point formula and Liebmann iteration, for the following mesh with uniform spacing and with boundary conditions as shown below in the figure. Obtain the results correct to two decimal places.  ***OR*** Solve by Crank-Nicolson method the following heat conduction equation $$u_t = u_{xx}$$ subject to $u(x,0) = 0, u(0,t) = 0$ and $u(1,t) = t$, for two time steps.	**20**	**CO4**