


Name:			
Enrolment No:			
UPES End Semester Examination, December 2023			
Program Name: BSc (Physics by Research)		Semester : VII	
Course Name: Monte Carlo Methods		Time : 3 hrs	
Course Code: PHYS 4025P		Max. Marks: 100	
Nos. of page(s): 2			
Instructions: Attempt all questions			
SECTION A (5Qx4M=20Marks)			
S. No.		Marks	CO
Q 1	Discuss the Buffon Needle problem. How does this problem illustrate the basic principles of Monte Carlo methods?	4	CO1
Q 2	Let G be the number of questions a student gets correct on a multiple-choice test with 10 questions, where the student guesses randomly on each question. Using Monte Carlo Method How will you estimate the mean number of correct answers?	4	CO2
Q 3	a. Define a Markov process. What are the key properties that distinguish it from other stochastic processes? b. Given a simple Markov chain with 3 states, construct the state transition matrix and interpret its meaning.	4	CO2
Q 4	Suppose that X and Y are independent binomial random variables with parameters (n, p) and (m, p). i. Find $E[e^{X+Y}]$ ii. Outline the MC procedure to $E[e^{X+Y}]$	4	CO2
Q 5	How MC Simulation will be useful in the context of Ising model?	4	CO3
SECTION B (4Qx10M= 40 Marks)			
Q 6	a. Provide an example of birth and death process in biological modeling. b. Describe the Chapman – Kolmogorov equation and explain how it can be used to study radioactivity. c. Discuss random walk problem. How will you simulate sample paths of the random walk. Outline the salient features of the algorithm.	10	CO1
Q7	a. Define a stochastic process. How do you classify it?	10	CO2

	<p>b. Define stationary and wide sense stationary process. Show that for a stationary stochastic process $X(t)$, $t \in T$, $E[X(t)]$ is constant. 2</p> <p>c. Define a Brownian motion process. What can you say about its correlation structure.</p> <p>d. Consider a stochastic process $X(t)$, $t \in T$ defined by $X(t) = a \cos (t+ \Phi)$ where a and ω are constants and Φ is a uniformly distributed random variable in the interval $[0, 2 \pi]$</p> <p>i. Simulate the sample paths.</p> <p>ii. How will you estimate the $E(X(t))$</p> <p>iii. Is $X(t)$ a wide-sense stationary stochastic process?</p>		
Q 8	Write pseudocode for the Metropolis-Hastings algorithm to sample from a given probability distribution. Include details on initializing the algorithm and determining convergence.	10	CO3
Q 9	<p>a. Discuss linear congruential method to generate random numbers? Why are such numbers called pseudo random numbers?</p> <p>b. Outline the procedure to generate exponentially distributed random numbers with parameter λ.</p> <p>c. Describe Box-Muller approach to generate the normal random variables.</p>	10	CO2/3
SECTION-C (2Qx20M=40 Marks)			
Q 10	<p>a. Show that the sum of independent identically distributed exponential random variables has a gamma distribution.</p> <p>b. Calculate the moment generating function of the uniform distribution on $(0, 1)$. Obtain $E[X]$ and $\text{Var}[X]$ by differentiating.</p> <p>c. State the central limit theorem. How will you illustrate this theorem using uniformly distributed random numbers.</p>	20	CO2
Q 11	<p>a. Define a Poisson process. What are the basic assumptions? Give two real life examples of the Poisson Process.</p> <p>b. Three friends A, B and C decide to meet at a certain place between 10:00 AM to 11:00 AM. Each will arrive randomly and wait for only 5 minutes. Using the Monte Carlo method, outline the procedure to estimate the probability of meeting.</p> <p>c. Explain the concept of transition probabilities in a Markov chain. How do they determine the behavior of the chain?</p>	20	CO3