Name: Enrolment No:	

UPES

End Semester Examination, December 2023

Course: **Advanced Engineering Mathematics-I** **Semester: I**

Program: **B. Tech. CSE** **Time : 03 hrs.**

Course Code: MATH1059 **Max. Marks: 100**

Instructions: Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (Each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 7 and 10 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Compute the partial derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ at the point $(1,2)$, where $u(x,y)=\log_e(x^2+y^2)$.	4	CO1
Q 2	Evaluate the integral $\int_0^2\int_{\sqrt{2x}}^2\left(\frac{y}{\sqrt{x^2+y^2+1}}\right)dydx$.	4	CO2
Q 3	Define divergence and curl of a vector point function.	4	CO3
Q 4	When a switch is closed in circuit containing a battery E, a resistor R and an inductance L, the current i builds up at a rate given by $$L\frac{di}{dt}+Ri=E.$$ Determine i as a function of t.	4	CO4
Q 5	Find the general solution of the differential equation $$(D^2+5D+6)y=0 \ (D \text{ stands for } \tfrac{d}{dx}).$$	4	CO4

SECTION B
(4Qx10M= 40 Marks)

Q 6	If $u=x+2y+z, v=x-2y+3z, w=2xy-xz+4yz-2z^3$, then find the Jacobian of u, v, w with respect to x, y, z.	10	CO1
Q 7	Change the order of integration and hence evaluate the integral $\int_0^a\int_0^y\left(\frac{x}{\sqrt{(a^2-x^2)(y-x)(a-y)}}\right)dxdy\ (a>0)$. **OR** Define Beta function. Using Beta and Gamma functions evaluate the integral $\int_{-1}^1(1-x^2)^n dx$, where n is a positive integer.	10	CO2

Q 8	Show that the following differential equation $$(x^4 - 2xy^2 + y^4)dx - (2x^2y - 4xy^3 + \sin y)dy = 0,$$ is exact and hence solve it.	**10**	**CO4**
Q 9	A competitive interaction is described by the Lotka-Volterra competition model $$x' = 0.01x(100 - x - y),$$ $$y' = 0.05y(60 - y - 0.2x).$$ Find all critical points of the system.	**10**	**CO5**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	(i) Find $curl(curl\,\bar{A})$, if $\bar{A} = x^2y\,\hat{\imath} - 2xz\,\hat{\jmath} + 2yz\,\hat{k}$ at the point $(1,0,2)$. (ii) Find the directional derivative of $\phi = xy^2 + yz^2$ at the point $(2,-1,1)$ in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. **OR** State Green's theorem. Verify Green's theorem for $\oint_C [(x^2 - 2xy)dx + (x^2y + 3)dy]$ where C is the boundary of the region bounded by the parabola $y = x^2$ and the line $y = x$.	**20**	**CO3**
Q 11	(i) Apply the method of variation of parameters to solve the following differential equation: $$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x + \cos x.$$ (ii) Find the general solution of the following differential equation: $$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = (2023)^x - \log_e(2024).$$	**10+10**	**CO4**