Name: Enrolment No:	

UPES

End Semester Examination, December 2023

Course: BCA **Semester: I**

Program: Basic Mathematics **Time : 03 hrs.**

Course Code: MATH 1058 **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Convert 278 into a binary number.	**4**	**CO1**
Q 2	Solve the quadratic equation $2x^2 + x - 528 = 0$.	**4**	**CO1**
Q 3	Find the polar coordinates where the cartesian coordinates are $(0, 1/2)$.	**4**	**CO3**
Q 4	Evaluate $\int sinx\ sin(cosx)\ dx$.	**4**	**CO2**
Q 5	Derive the equation of tangent at (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	i) If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots to\ \infty}}}}$, then prove that $\frac{dy}{dx} = \frac{\cos x}{(2y-1)}$. ii) If $z = x^4 + y^4 + 3x^2y^2$, then find the value of $\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$.	**10**	**CO2**
Q 7	Calculate the differential coefficient of (i) e^{sinx^2} (ii) $\log sinx^2$ with respect to x.	**10**	**CO2**
Q 8	Given $\mathcal{R} = t^m A + t^n B$, where A, B are constant vectors, show that, if $\mathcal{R}$ and $\frac{d^2\mathcal{R}}{dt^2}$ are parallel vectors, then $m + n = 1$, unless $m = n$.	**10**	**CO3**
Q 9	Find the area of a plate in the form of a quadrant of the circle $x^2 + y^2 = a^2$. **OR** Evaluate the integral $\int \frac{3x+5}{x^3 - x^2 - x + 1}\, dx$.	**10**	**CO2**

SECTION-C (2Qx20M=40 Marks)			
Q 10	A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 35 per package of nuts and ₹ 14 per package of bolts. How many packages of each should be produced each day to maximize his profit, if he operates each machine for at most 12 hours a day? Convert it into a linear programming problem and solve graphically.	**20**	**CO4**
Q 11	a) Under what condition the straight-line $y = mx + c$ may be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. b) Evaluate the volume of a parallelepiped whose coterminous edges are $\hat{\imath}-\hat{\jmath}+\hat{k}$, $2\hat{\imath}+3\hat{\jmath}-\hat{k}$, and $-\hat{\imath}-\hat{\jmath}+5\hat{k}$. **OR** a) Under what condition the straight-line $y = mx + c$ may be a tangent to the parabola $y^2 = 4ax$. b) Let $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$, $\vec{b}=\hat{\imath}$ and $\vec{c}=c_1\hat{\imath}+c_2\hat{\jmath}+c_3\hat{k}$. If $c_1=1$ and $c_2=2$, find c_3 such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.	**20**	**CO3**