Name:

Enrolment No:

UPES

End Semester Examination, May 2024

Course: Linear Algebra-II **Semester: II**
Program: B.Sc. (Hons.) Math by Research **Time : 03 hrs.**
Course Code: MATH1063 **Max. Marks: 100**

Instructions: 1. Attempt all the questions.
2. All the mathematical symbols have their usual meaning.
3. Attempt one question between the internal choices give in question 7 and question 10.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	For what value of b is the following matrix A positive definite. $$A = \begin{bmatrix} 2 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 2 \end{bmatrix}$$	**4**	**CO1**
Q 2	Consider the following two bases of $R^2(R)$: $S = \{u_1, u_2\} = \{(1,2), (3,5)\}$ and $S' = \{v_1, v_2\} = \{(1,-1), (1,-2)\}$ Find the transition matrix $P_{S\to S'}$ from S to S'. Then, calculate transition matrix $P_{S'\to S}$ using $P_{S\to S'}$.	**4**	**CO2**
Q 3	If $V(F)$ is a finite dimensional vector space and W is a subspace of $V(F)$ then prove that $$\dim(W) + \dim(W^o) = \dim(V)$$ where, W^o denotes the annihilator of W.	**4**	**CO3**
Q 4	Find the closest point to the vector $x = (3,1,5,1)$ in the subspace W spanned by $v_1 = (3,1,-1,1)$ and $v_2 = (1,-1,1,-1)$. Also, calculate the vector which is orthogonal to each vector in W.	**4**	**CO4**
Q 5	Consider the following polynomials in $P(t)$ with inner product $$< f, g > = \int_0^1 f(t)g(t)dt$$ $f(t) = t + 2, g(t) = 3t - 2, h(t) = t^2 - 2t - 3$. Then, find $< f, g >$ and $< f, h >$. Also, normalize f and g.	**2+2**	**CO4**

SECTION B (4Qx10M= 40 Marks)			
Q 6	Find the Jordan canonical form J of matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$. Also, find matrix S such that $A = SJS^{-1}$, where J is the Jordan canonical form of matrix A.	**10**	**CO1**
Q 7	Let $\phi: G \to G'$ be a group homomorphism. Then prove that Kernel $\ker(\phi)$ is a normal subgroup of G and $\frac{G}{\ker(\phi)} \simeq Im(\phi)$ where, $\simeq$ denotes the group isomorphism. OR State and prove second isomorphism theorem.	**10**	**CO2**
Q 8	If the ordered basis of $P_2(R)$ is $\{1, 1+x, 1+x+x^2\}$ with respect to which the basis of its dual space is $\{f_1, f_2, f_3\}$ and $f: P_2(R) \to R(R)$ is linear functional given by $f(a+bx+cx^2) = 2a+3b-7c$ and if $f = c_1 f_1 + c_2 f_2 - 2c_3 f_3$ then find the values of c_1, c_2, c_3.	**10**	**CO3**
Q 9	State and prove the Bessel's inequality.	**10**	**CO4**
SECTION-C (2Qx20M=40 Marks)			
Q 10	a. Define the transpose of a linear transformation. Let ϕ be a linear functional on R^2 defined by $\phi(x,y) = x - 2y$. For the linear mapping $T: R^2 \to R^2$ given by $T(x,y) = (y, x+y)$, find $[T^t(\phi)](x,y)$. b. Let $T: P_3(R) \to P_3(R)$ is a linear operator given by $$T\big(p(x)\big) = p(x+1)$$ where, $P_3(R)$ denotes the polynomial with real coefficients of degree at most 3. Find the eigenvalues and eigenvectors of linear operator T. Also, calculate the characteristic and minimal polynomial of T. Is this linear operator diagonalizable? **OR** a. Define the transpose of a linear transformation. Let ϕ be a linear functional on R^2 defined by $\phi(x,y) = x - 2y$. For the linear mapping $T: R^2 \to R^2$ given by $T(x,y) = (2x-3y, 5x+2y)$, find $[T^t(\phi)](x,y)$. b. Let $T: P_4(R) \to P_4(R)$ is a linear operator given by	**5+15**	**CO3**

	$$T(p(x)) = p'''(x)$$ where, $p'''(x)$ is the third derivative of polynomial $p(x)$ and $P_4(R)$ denotes the polynomial with real coefficients of degree at most 4. Find the eigenvalues and eigenvectors of linear operator T. Also, calculate the characteristic and minimal polynomial of T. Is this linear operator diagonalizable?		
Q 11	Fit a least square regression line to the following data using least square approximation. X: 1, 2, 4, 6, 8 Y: 3, 4, 8, 10, 15	**20**	**CO4**

X	1	2	4	6	8
Y	3	4	8	10	15