

<b>Name:</b> <b>Enrolment No:</b>	
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<b>UPES</b> <b>End Semester Examination, May 2024</b>	<b>Semester: II</b> <b>Time: 03 hrs.</b> <b>Max. Marks: 100</b>
<b>Course: Calculus</b> <b>Program: B.Sc. (H) Physics by Research</b> <b>Course Code: MATH1030</b>	
<b>Instructions: Attempt all questions.</b>	

<b>SECTION A</b> <b>(5Qx4M=20Marks)</b>			
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S. No.		Marks	CO
Q 1	If $u = (1 - 2xy + y^2)^{-\frac{1}{2}}$ , then show that $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = y^2 u^3$ .	4	CO1
Q 2	Obtain the value of $y_n$ when $y = \cos^2 x \cdot \sin^3 x$ .	4	CO1
Q 3	If $x = t - \frac{t^3}{3}$ , $y = t^2$ , then find radius of curvature at origin.	4	CO2
Q 4	Evaluate $\int_0^\infty \int_0^\infty e^{-x^2(1+y^2)} x \, dx \, dy$ .	4	CO3
Q 5	Use gamma function to evaluate $\int_0^\infty \sqrt[4]{x} e^{-\sqrt{x}} dx$ .	4	CO4

<b>SECTION B</b> <b>(4Qx10M= 40 Marks)</b>			
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Q 6	If $y = [x + \sqrt{1 + x^2}]^m$ , then apply Leibnitz theorem to prove that $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$ .	10	CO1
Q 7	Find all the asymptotes of the curve $2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$ .	10	CO2
Q 8	Using the transformation $x + y = u$ and $y = uv$ , evaluate $\iint [xy(1 - x - y)]^{1/2} dx dy$ , integration being taken over the area of the triangle bounded by the lines $x = 0, y = 0, x + y = 1$ .	10	CO3
Q 9	Trace the curve of $y^2(a - x) = x^2(a + x)$ . <b>OR</b> Prove that if the perimeter of a triangle is constant, its area is maximum when the triangle is equilateral.	10	CO2

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**SECTION-C**  
**(2Qx20M=40 Marks)**

Q 10	Change the order of integration in $I = \int_0^{4a} \int_{x^2/4a}^{2-x} dy dx$ and hence evaluate.	<b>20</b>	<b>CO3</b>
Q 11	If $u^3 + v^3 + w^3 = x + y + z$ , $u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ , $u + v + w = x^2 + y^2 + z^2$ , then prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$ . <b>OR</b> Find the volume of the region bounded by the surface $y = x^2$ , $x = y^2$ and the planes $z = 0$ , $z = 3$ .	<b>20</b>	<b>CO4</b>

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