Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2024

Course: Real Analysis I **Semester: II**

Program: B.Sc. (Mathematics by Research) **Time: 03 hrs.**

Course Code: MATH1068 **Max. Marks: 100**

Instructions: Attempt all questions from Section A, Section B and Section C. There are internal choices in Questions 7 and 10.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Show that the set $S = \left\{1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, \dots\right\}$ is neither open nor closed.	**4**	**CO1**
Q 2	Find the supremum and infimum, if they exist, of the set $x \in \mathbb{Q}: x = \frac{n}{n+1}, n \in \mathbb{N}$.	**4**	**CO1**
Q 3	Identify the following sequence is bounded or not bounded: $\left\{1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}\right\}$	**4**	**CO2**
Q 4	Evaluate $\lim_{x \to \pi} \frac{\sin x}{2 + \cos x}$.	**4**	**CO3**
Q 5	Does the function $f: [0,2] \to \mathbb{R}$ defined by $f(x) = x^2$ satisfy the Lipschitz condition?	**4**	**CO3**

SECTION B

(4Qx10M= 40 Marks)

Q 6	If $0 < \theta < 1$, $\lvert x \rvert < 1$, show that $\left\lvert \frac{x(1-\theta)}{1+\theta x} \right\rvert < 1$.	**10**	**CO1**
Q 7	Determine whether the following sequence is non decreasing and bounded from above? $$\{a_n\} = \left\{\frac{2^n 3^n}{n!}\right\}$$ If it is convergent, then find the limit of the convergent sequence. **OR** Show that the sequence $\{s_n\}$ defined by the formula $s_1 = 1$, $s_{n+1} = \sqrt{(3s_n)}$ converges to 3.	**10**	**CO2**

Q 8	Discuss the existence of the limit of the function f defined as $$f(x)=\begin{cases}1, & if\ x<1\\ 2-x, & if\ 1<x<2\\ 2, & if\ x\geq 2\end{cases}$$ at $x=1$ and $x=2$.	**10**	**CO3**
Q 9	Determine the values of a, b, c for which the function $$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x} & for\ x<0\\ c & for\ x=0\\ \frac{(x+bx^2)^{\frac{1}{2}}-x^{\frac{1}{2}}}{bx^{\frac{3}{2}}} & for\ x>0\end{cases}$$ is continuous at $x=0$	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	State and prove Cauchy's second theorem on limits. **OR** Show that necessary and sufficient condition for the convergence of monotonic sequence is that it is bounded.	**20**	**CO2**
Q 11	(i) Check whether the function $f:[-2,2]\to\mathbb{R}$ defined by $f(x)=x^3$ is uniformly continuous or not? (ii) Let $y=E(x)$, where $E(x)$ denotes the integral part of x. Prove that the function is discontinuous where x has an integral value. Also draw the graph.	**20**	**CO3**