Name: **Enrolment No:**	

UPES

End Semester Examination, May 2024

Course: Probability & Statistics — **Semester: IV**

Program: B.Sc. (Hons.) Mathematics — **Time : 03 hrs.**

Course Code: MATH2052 — **Max. Marks: 100**

Instructions: Attempt all questions from Section A, B and C. Questions 9 and 10 have internal choices. Certain abbreviations are conventionally aligned as discussed in classes.

SECTION A

(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	For each of the following random variables, find the moment generating function (MGF). a) X is a discrete random variable, with probability mass function (PMF) $p_X(k)=\begin{cases}\frac{1}{3}, k=1\\ \frac{2}{3}, k=2\end{cases}$ b) Y is $Uniform(0,1)$ random variable.	**4**	**CO1**
Q 2	If $X_1, X_2, \ldots, X_n$ are n independent random variables, then show that $\phi_{X_1+X_2+\ldots+X_n}(\omega)=\phi_{X_1}(\omega)\phi_{X_2}(\omega)\cdots\phi_{X_n}(\omega)$, where $\phi_{(.)}(\omega)$ is the characteristic function (CF) of the corresponding random variable $(\cdot)$.	**4**	**CO1**
Q 3	You go to the gym. The speed X of the treadmill is uniformly distributed between 5 to $10\,km/hr$. Find the probability density function (PDF) of the time it takes to run $10\,km$.	**4**	**CO2**
Q 4	Define Correlation coefficient, $\rho(X,Y)$,of two random varibales X and Y. Further show that $-1 \le \rho(X,Y) \le 1$.	**4**	**CO4**
Q 5	Show that second order moment of a random variable X is minimum when taken about its mean.	**4**	**CO3**

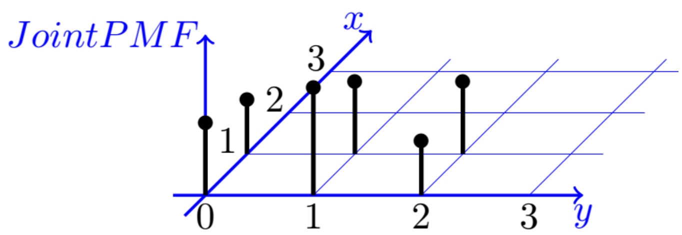
SECTION B

(4Qx10M= 40 Marks)

Q 6	If $X\,Exponential(\lambda)$,then find the mean and variance of the random variable X. What is the probability that the random variable X is less that its expected value?	**10**	**CO2**
Q 7	Let X_1 and X_2 be two independent random variables having mean $\mu=0$ and variance $\sigma^2=16$. Compute the probability $P(X_1^2+X_2^2>8)$.	**10**	**CO2**
Q 8	Consider two random variables X and$Y$ with joint Probability mass function (PMF) given in the following table:	**10**	**CO3**

	$Y=0$	$Y=1$	$Y=2$
$X=0$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
$X=1$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

The following figure shows joint PMF, $p_{XY}(x,y)$.



a) Find $P(X=0, Y\leq 1)$.
b) Find the marginal PMFs of X and Y.
c) Find $P(Y=1|X=0)$.
d) Are X and Y independent?

Q 9 | **10** | **CO1**

Let X Be a standard normal variate. Find the PDF of Y, where $Y=\frac{1}{2}X^2$.

OR

Let X be a continuous random variable and let $f_X(x)$ be the corresponding PDF. Also let $y=g(x)$ be a continuously differentiable function for all values of x. If $f_Y(y)$ be the PDF of the random variable Y, given by $Y=g(X)$ and if $\frac{dy}{dx}$ is either positive or negative for all x, then show that $f_Y(y)=f_X(x)\left|\frac{dx}{dy}\right|$, where $y\in$ range of g and where you can assume that $f_X(x)$ is defined for all values of x.

SECTION-C
(2Qx20M=40 Marks)

Q 10 | **20** | **CO4**

Answer the following.

a) State and proof Chebyshev's inequality.
b) State central limit theorem. Suppose a person loads 100 packages onto a plane, each package's weight being an independent random variable uniformly distributed between 5 and 50 pounds. What is the likelihood that the total weight exceeds 3000 pounds?

Suppose we poll n voters and record the fraction M_n of those polled who are in favor of a particular candidate. If p is the fraction of the entire voter population that supports this candidate, then find the probability $P(|M_n - p|\geq\epsilon)$ that the polling error is larger than some desired accuracy

	ϵ. Also find how large a sample size n is needed if we wish our estimate M_n to be within 0.01 of p with probability at least 0.95?		
Q 11	Derive the normal equations in the least square method when fitting a straight line from a given sample size of n data, $(x_i, y_i), i=1,2,\ldots,n$. And hence fit the line based on the following sample: x: 5, 10, 15, 20, 25 y: 16, 19, 23, 26, 30	**20**	**CO4**

x	5	10	15	20	25
y	16	19	23	26	30