Name:

Enrolment No:

UPES

End Semester Examination, May 2024

Course: **Advanced Engineering Mathematics-II** **Semester : II**

Program: **B. Tech. CSE** **Time : 03 hrs.**

Course Code: **MATH1065** **Max. Marks: 100**

Instructions: Attempt all questions from Sections **A, B,** and **C**. Questions **6** and **11** have internal choices**.**

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	The values of a function $f(x)$ at four discrete points are as follows: x: 0, 1, 3, 4 $f(x)$: -12, 0, 6, 12 If the function $f(x)$ may be represented by a polynomial $f(x) = (x-a)R(x)$, where $R(x)$ is a polynomial of degree 2, obtained by Lagrange's interpolation formula and a is a real constant. Find a and the polynomial $R(x)$.	**4**	**CO1**
Q 2	Expand $f(z) = \frac{1}{z(z-2)}$ as a Laurent's series for the region $\|z\| > 2$.	**4**	**CO2**
Q 3	Discuss the nature of the singularity of the function $f(z) = \frac{\sin(z-a)}{(z-a)}$ at $z = a$.	**4**	**CO2**
Q 4	Compute the Laplace transform of the function $f(t) = t^{\frac{3}{2}} - 2^t$.	**4**	**CO4**
Q 5	Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 5\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$.	**4**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

Q 6	A curve $y = f(x)$ is drawn to pass through the points given by the following table: x: 1, 1.5, 2, 2.5, 3, 3.5, 4 y: 2, 2.4, 2.7, 2.8, 3, 2.6, 2.1 Find the area bounded by the curve $y = f(x)$, the straight lines $x = 1, x = 4$ and the x-axis using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule.	**10**	**CO1**

	OR Upon completing the construction of his house Dr. Joshi discovers that 100 square feet of plywood scrap and 80 square feet of white-pine scrap are in usable form for the construction of tables and bookcases. It takes 16 square feet of plywood and 8 square feet of white-pine to make a table; 12 square feet of plywood and 16 square feet of white-pine are required to construct a bookcase. By selling the finished products to a local furniture store, Dr. Joshi can realize a profit of Rs. 25 on each table and Rs. 20 on each bookcase. How many he most profitably use the leftover wood? Use Simplex method to solve the problem. *Problem Formulation*: $$Max.z = 25x_1 + 20x_2$$ such that $$16x_1 + 12x_2 \leq 100$$ $$8x_1 + 16x_2 \leq 80$$ and $$x_1, x_2 \geq 0.$$ Here, x_1, x_2 and z denotes number of tables, number of book cases and total profit respectively.		
Q 7	Evaluate the integral $\int_C \frac{f(z)}{g(z)}\,dz$, where $f(z) = e^{ez}$, $g(z) = z - ei$ and $C:\ |z-2| + |z+2| = 6$. (Here $i = \sqrt{-1}$)	**10**	**CO2**
Q 8	Find the singular points of the following differential equation and classify them. $$x^3(x-2)\frac{d^2y}{dx^2} + x^3\frac{dy}{dx} + 6y = 0.$$	**10**	**CO3**
Q 9	State second shifting theorem. Express the following function $f(t)$ in terms of Unit step function and hence find its Laplace transform. $$f(t) = \begin{cases} 2, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \\ \sin t, & t > 2\pi \end{cases}$$	**10**	**CO4**
	SECTION-C **(2Qx20M=40 Marks)**		
Q 10	**(i)** A voltage Ee^{-at} is applied at $t = 0$ to a circuit of inductance L and resistance R. The equation governing the current flow in LR circuit is given by $$L\frac{dI}{dt} + RI = Ee^{-at}\ .$$ Using Laplace transform, show that the current at any time t is given by $$\frac{E}{R-aL}\left(e^{-at} - e^{-Rt/L}\right).$$ **(ii)** Find the Fourier series expansion of the function $f(x) = \sqrt{1-\cos x}$ in the interval $(0, 2\pi)$.	**10+10**	**CO4**
Q 11	If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b\sin^3\left(\frac{\pi x}{l}\right)$ where b is a constant, find the displacement $y(x,t)$.	**20**	**CO5**

	OR A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position then find the displacement $y(x, t)$.		