Name:

Enrolment No:

UPES

End Semester Examination, May 2024

Course: Mathematical Sciences II | **Semester: II**

Program: B. Sc CS | **Time: 03 hrs.**

Course Code: MATH1067 | **Max. Marks: 100**

Instructions: Attempt all questions

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Test the analyticity of the function $f(z) = z\|z\|$.	**4**	**CO2**
Q 2	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(n-\log n)^n}{2^n.n^n}$.	**4**	**CO2**
Q 3	Find the Laplace transform of $\frac{\cos 2t \sin t}{e^t}$.	**4**	**CO3**
Q 4	Expand $f(x) = x$ as a half-range sine series in the interval $0 < x < 2$.	**4**	**CO3**
Q 5	Classify the following partial differential equations: (i) $4\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial t^2} = 0,$ (ii) $x^2\frac{\partial^2 u}{\partial t^2} + 3\frac{\partial^2 u}{\partial x \partial t} + x\frac{\partial^2 u}{\partial x^2} + 17\frac{\partial u}{\partial t} = 100u.$	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Expand $f(z) = \frac{(z+1)}{(z-3)(z-4)}$ as a Taylor's series about $z = 2$.	**10**	**CO2**
Q 7	Using convolution theorem to find the inverse Laplace transform of $\frac{1}{(p^2+a^2)(p^2+b^2)}$.	**10**	**CO3**
Q 8	Evaluate $\int_0^{1+i}(x - y + ix^2)dz$ along the line from $z = 0$ to $z = 1 + i$.	**10**	**CO2**
Q 9	Find the Fourier series of $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$. **OR** Solve $\frac{dy}{dt} + y = \cos 2t, y(0) = 1$ using Laplace transformation.	**10**	**CO3**

SECTION-C (2Qx20M=40 Marks)			
Q 10A	Use Picard's method to solve $\frac{dy}{dx} = 1 + xy$ with $x_0 = 0, y_0 = 0$ up to third approximation.	**10**	**CO1**
Q 10B	Apply Newton-Raphson method to find a root of $x^4 - x - 10 = 0$, correct to three decimal places.	**10**	**CO1**
Q 11A	Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y} + x^3$. **OR** Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial y} - 1 = \cos(x + 2y) + e^y$.	**10**	**CO4**
Q 11B	Apply method of separation of variable to solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, given that $u(0, y) = 8e^{-3y}$. **OR** Solve the one-dimensional heat equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod without radiation subject to the following conditions: (i) u is finite when $t \to \infty$. (ii) $u = 0$ when $x = \ell$ for all values of t. (iii) $\frac{\partial u}{\partial x} = 0$ when $x = 0$ for all values of t. (iv) $u = u_0$ when $t = 0$ for $0 < x < 1$.	**10**	**CO4**

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