Name: **Enrolment No:**	

UPES

End Semester Examination, December 2024

Course: Topics in Mathematical Sciences
Semester: 1
Program: MCA **Time: 03 hrs.**
Course Code: CSEG7026 **Max. Marks: 100**

Instructions:
1) All questions are compulsory
2) Use of calculator is allowed

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	State the properties of entropy.	**4**	**CO4**
Q 2	Explain how Monte Carlo integration works and why it is particularly useful in high-dimensional problems.	**4**	**CO3**
Q 3	What is a Hidden Markov Model (HMM)? Outline its primary components and give an example of a situation where it might be used.	**4**	**CO4**
Q 4	Describe the differences between an M/M/1 and an M/M/m queuing system.	**4**	**CO5**
Q 5	Differentiate between a discrete random variable and a continuous random variable. Give an example of each	**4**	**CO2**
SECTION B **(4Qx10M= 40 Marks) – Attempt any 4**			
Q 6	a. Explain the process of generating random numbers for Monte Carlo simulations. b. Discuss the importance of pseudo-random number generators and their limitations in simulations.	**10**	**CO4**
Q 7	Explain the classification of stochastic processes in detail. Discuss the differences between discrete and continuous processes, giving examples of each.	**10**	**CO1**
Q 8	Define a random variable and explain the difference between probability mass function (PMF) and probability density function (PDF). Illustrate with examples of a discrete and a continuous random variable, explaining how to calculate probabilities for each type.	**10**	**CO2**
Q 9	Explain Little's Law and its application in queuing systems. Provide an example demonstrating its use in a practical situation.	**10**	**CO3**
Q 10	Provide a brief definition of KL divergence and describe its purpose in comparing two probability distributions.	**10**	**CO4**

SECTION-C
(2Qx20M=40 Marks) – Attempt any 2

<table>
<tr><td>Q 11</td><td>A company receives an average of 5 customer calls per hour. Assume that the customer calls follow a Poisson process.
a) What is the probability that the company will receive exactly 3 calls in an hour?
b) What is the probability that the company receives no calls in a 30-minute period?
c) Calculate the expected number of calls the company will receive in a 3-hour period.</td><td></td><td>CO5</td></tr>
<tr><td>Q 12</td><td>Consider a discrete random variable X with the following probability distribution:

<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>0.1</td><td>0.3</td><td>0.4</td><td>0.2</td></tr></table>
1. Calculate the expected value (mean) E(X) of the random variable X.
2. Find the moment-generating function (MGF) $M_X(t)$ of the random variable X.
3. Use the MGF to find the second moment $E(X^2)$ of X and use it to compute the variance Var(X).</td><td></td><td>CO5</td></tr>
<tr><td>Q 13</td><td>A call center uses two different systems to manage incoming calls:

i. System 1: An M/M/m system with 3 service agents.
ii. System 2: An M/M/m/m system with 3 service agents and a maximum queue length of 3.

The arrival rate of calls is 18 calls per hour, and each agent can handle 10 calls per hour.

A. For System 1 (M/M/m), calculate the following:
a) The probability that there are 0 calls in the system.
b) The average number of calls in the system (L).
c) The average waiting time in the system (W).

B. For System 2 (M/M/m/m), calculate the following:
a) The probability that all servers are busy.
b) The probability that a customer has to wait in the queue.</td><td></td><td>CO5</td></tr>
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