Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: **Advanced Engineering Mathematics-I** **Semester: I**

Program: **B. Tech. SoCS** **Time : 03 hrs.**

Course Code: MATH1059 **Max. Marks: 100**

Instructions: Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 7 and 10 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Define homogeneous function. Check whether the following function $$u(x,y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log_e x - \log_e y}{x^2+y^2} \ (x > 0,\ y > 0)$$ is homogeneous or not.	**4**	**CO1**
Q 2	If $z = e^{ax+by}$, compute $\left(b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y}\right)$ where a and b are constants.	**4**	**CO1**
Q 3	Evaluate the triple integral $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)dydxdz$.	**4**	**CO2**
Q 4	What is the greatest rate of increase of $u(x,y,z) = xyz^2$ at $(1,0,3)$?	**4**	**CO3**
Q 5	Find the general solution of the differential equation $$(D^2+6D+9)y = 0 \ (D \text{ stands for } \tfrac{d}{dx}).$$	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	If $y(x) = \sin ax + \cos ax$ then show that $$y_n = a^n\sqrt{1+(-1)^n \sin 2ax}\,,$$ where y_n denotes the n^{th} derivative of y with respect to x.	**10**	**CO1**
Q 7	Find the area of the curvilinear triangle bounded by the parabolas $y^2 = 12x$, $x^2 = 12y$ and the circle $x^2+y^2 = 45$ which lies outside the circle. **OR** If $m, n, a, b > 0$ then by using Beta function, prove that: $$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}dx = \frac{\beta(m,n)}{(a+b)^m a^n}.$$	**10**	**CO2**

Q 8	Show that the following differential equation $$(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0,$$ is exact and hence solve it.	**10**	**CO4**
Q 9	Find and classify the critical points of the following plane autonomous system as stable or unstable. $$x'(t) = x^2 + y^2 - 6,$$ $$y'(t) = x^2 - y.$$	**10**	**CO5**
SECTION-C (2Qx20M=40 Marks)			
Q 10	(i) The velocity vector field of an ideal fluid is given by $$\overrightarrow{V}(x, y, z) = (y + z)\hat{\imath} + (z + x)\hat{\jmath} + (x + y)\hat{k}.$$ Show that $\overrightarrow{V}$ is irrotational and incompressible. (ii) Find the work done in moving a particle along the straight-line segments joining the points (0,0,0) to (1,0,0), then to (1,1,0) and finally to (1,1,1) under the force field $\overrightarrow{F} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$. **OR** Verify Green's theorem in the plane for $$\oint_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy],$$ where C is the boundary described counter-clockwise of the triangle formed by the lines $x = 0$, $y = 0$ and $x + y = 1$.	**20**	**CO3**
Q 11	(i) If x^n is an integrating factor of the differential equation $$(y - 2x)^3dx - x(1 - xy)dy = 0.$$ Then find n and hence solve the equation. (ii) Find the general solution of the following differential equation: $$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{-x} + 2\cos(2x + 3).$$	**10+10**	**CO4**