Name:

Enrolment No:

UPES

End Semester Examination, December 2024

Course: Basic Mathematics **Semester: I**

Program: BCA **Time: 03 hrs.**

Course Code: MATH1058 **Max. Marks: 100**

Instructions: Attempt all questions. Calculator is allowed.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	If $u = \frac{x^3+y^3}{x^2+y^2}$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	**4**	**CO2**
Q 2	Find the Compound Interest on Rs 48000 for 1 years at 8% per annum when interest is compounded half-yearly.	**4**	**CO1**
Q 3	If $x = -\frac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.	**4**	**CO1**
Q 4	Find the center and radius the circle $x^2 + y^2 - 8x + 10y - 12 = 0$.	**4**	**CO3**
Q 5	Three metal cubes of edge lengths 3 cm, 4 cm and 5 cm are melted to form a single cube. Find the edge of such cube.	**4**	**CO1**

SECTION B
(4Qx10M= 40 Marks)

Q 6	If $y = e^{x+e^{x+e^{x+\infty}}}$ then find first derivative of y with respect to x.	**10**	**CO2**
Q 7	If $u = e^{xyz}$ then show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$.	**10**	**CO2**
Q 8	Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.	**10**	**CO3**
Q 9	Evaluate the definite integral $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$. **OR** Evaluate $\iint (x^2 - y^2)\, dxdy$ over the triangle with vertices (0,1), (1,1) and (1,2).	**10**	**CO2**

SECTION-C (2Qx20M=40 Marks)			
Q 10	(i) Find the value of λ so that the two vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = -4\hat{\imath} - 6\hat{\jmath} + \lambda\hat{k}$ are perpendicular to each other. (ii) Show that the points with the position vectors $\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}, \vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}, \vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ then Prove that $\vec{a}, \vec{b}, \vec{c}$ are coplanar. **OR** (i) Find the equation of a line passing through the point $(3, -2)$ and perpendicular to the line $x - 3y + 5 = 0$. (ii) Find the coordinates of the foci, the vertices, the lengths of major and minor axes and the eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$.	**20**	**CO3**
Q 11	A manufacturer makes Rs 600 profit on each $21''$ TV set it produces and Rs 400 profit on each $14''$ TV set. A $21''$ TV requires 1 hour on machine X, 1 hour on machine Y and 4 hours on machine Z. The $14''$ TV requires 2 hours on machine X, 1 hour on machine Y and 1 hour on machine Z. In a given day machine X, Y, Z can work a maximum of 16, 9 and 24 hours respectively. (i) Formulate this problem as a linear programing problem so that he maximizes his profit. (ii) Draw the feasible region on a graph and clearly mention the corner points. (iii) Solve this problem graphically and find, how many $21''$ TV sets and how many $14''$ TV sets should produce per day to maximize the profit.	**20**	**CO4**

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