


Name:			
Enrolment No:			
UPES End Semester Examination, Dec 2024			
Course: Mathematical Science – 1 Program: B.Sc. CS Course Code: MATH-1060		Semester: I Time: 03 hrs. Max. Marks: 100	
Instructions: Read all the below mentioned instructions carefully and follow them strictly: 1) Mention Roll No. at the top of the question paper. 2) Attempt all the parts of a question at one place only.			
SECTION A (5Qx4M=20Marks)			
S. No.		Mar ks	CO
Q 1	State Rolle's theorem and explain the geometric interpretation of it.	4	CO1
Q 2	If $u = (1 - 2xy + y^2)^2$, then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.	4	CO1
Q 3	Verify Lagrange's mean value theorem for the function $f(x) = e^x$ in the interval $[0, 1]$.	4	CO1
Q 4	Evaluate $\iint_R (x + y) dy dx$, where R is the region bounded by the lines $x = 0$, $x = 2$, $y = x$ and $y = x + 2$.	4	CO2
Q 5	Define forward difference operator (Δ). Prove that $\Delta^2 y_0 = y_2 - 2y_1 + y_0$.	4	CO4
SECTION B (4Qx10M= 40 Marks)			
Q 6	Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ along the parabola $y^2 = x$ between the points $(0, 0)$ to $(1, 1)$ where $\vec{F} = x^2\hat{i} + xy\hat{j}$ and $\vec{r} = x\hat{i} + y\hat{j}$.	10	CO2
Q 7	Define exact differential equation. Find the values of constant λ such that $(x^2 - \lambda xy - 2y^2)dx + (y^2 - 4xy - 2y^2)dy = 0$ is exact. Further, for this value of λ , solve the given differential equation.	10	CO3
Q 8	What is the main difference between a difference equation and a differential equation? Solve the difference equation: $y_{n+3} - 2y_{n+1} + 4y_n = 0$.	10	CO4

Q 9	<p>If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in xy plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$. Find the work done.</p> <p style="text-align: center;">OR</p> <p>(a) Find the constant a such that $(3x - 2y + z)\hat{i} + (4x + ay - z)\hat{j} + (x - y + 2z)\hat{k}$ is solenoidal.</p> <p>(b) Show that the vector $(6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.</p>	10	CO2
<p>SECTION-C (2Qx20M=40 Marks)</p>			
Q 10	<p>Define Wronskian. Evaluate Wronskian of the functions $y_1(x) = e^x$ and $y_2(x) = e^{3x}$. Show that e^x and e^{3x} are linearly independent solutions of second order linear differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.</p>	20	CO3
Q 11	<p>(a) Explain the different cases of the roots of the characteristic equation for a second order linear homogeneous ordinary differential equation with constant coefficient. Find the general solution when the roots are distinct, repeated, or complex?</p> <p>(b) Obtain the complementary function (C.F.) and particular integral (P.I) of the differential equation: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$, given $y = 3$ and $\frac{dy}{dx} = 3$ when $x = 0$.</p> <p style="text-align: center;">OR</p> <p>Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + a^2y = \operatorname{cosec}(ax)$.</p>	20	CO3